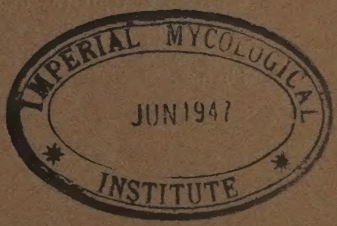
COMPTES RENDUS
(DOKLADY)
DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1943



VOLUME XLI

№ 2

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

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11-me année

Paraissant tous les dix jours

1943

VOLUME XLI, № 2

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MATHEMATICS

HERMITE'S CRITERION FOR INTEGRAL FUNCTIONS OF EXPONENTIAL TYPE. I

By B. LEVIN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 20. XII. 1942)

Hermite gave the following criterion that all roots of a polynomial lie in the upper semi-plane. In order that all roots of the polynomial

$$f(x)=P(x)+iQ(x),$$

where $P(x)$ and $Q(x)$ are real, lie in the upper semi-plane, it is necessary and sufficient that all roots of $P(x)$ and $Q(x)$ be real, alternate and that

$$Q'(x)P(x)-P'(x)Q(x)>0 \qquad (1)$$

in at least one point of the real axis.

This criterion can be extended to integral functions of genus 0 (1).

In the papers of N. Tschebotarëw (2) and L. Pontrjagin (3) the Hermite criterion was extended to some special classes of integral functions of exponential type. In the present note we solve the problem of applicability of the Hermite criterion to an arbitrary integral function of exponential type. We have the following fundamental

Theorem 1. *The integral function of exponential type* $f(x)=Px)+iQ(x)$, *where* $P(x)$ *and* $Q(x)$ *are real, satisfies the conditions:*

(A) *all the roots* α_k *of the function* $f(z)$ *lie in the upper semi-plane, i. e.,* $I\alpha_k\geqslant 0$;

(B) *the type of* $f(z)$ *on the ray* $\arg z=\frac{\pi}{2}$ *does not exceed that on the ray* $\arg z=-\frac{\pi}{2}$, *i. e.* $h\left(\frac{\pi}{2}\right)\leqslant h\left(-\frac{\pi}{2}\right)$, *where* $h(\varphi)$ *is the indicatrix of growth of* $f(z)$, *if and only if the following conditions are fulfilled:*

(C) *all roots of* $P(x)$ *and* $Q(x)$ *are real and they alternate, i. e.* $P(x)=Ae^{ax}\prod_{-\infty}^{\infty}\left(1-\frac{x}{a_k}\right)e^{\frac{x}{a_k}}$ *and* $Q(x)=Be^{bx}\prod_{-\infty}^{\infty}\left(1-\frac{x}{b_k}\right)e^{\frac{x}{b_k}}$* $(a_k\leqslant b_k\leqslant a_{k+1})$;

(D) $a-b+\sum_{-\infty}^{\infty}\left(\frac{1}{a_k}-\frac{1}{b_k}\right)=0.$

Let us first prove some lemmas.

Lemma 1. *If the integral function* $f(z)$ *of exponential type satisfies the condition* (A), *then* $\sum_{-\infty}^{\infty}-I\left(\frac{1}{z_k}\right)<\infty$.

* The assumption $P(0)\neq 0$, $Q(0)\neq 0$ does not impose any restriction, because the general case may be reduced to this particular one by means of a translation.

This lemma follows immediately from Lindelöf's theorem (4). For the integral function of exponential type $\left|\sum_{|\alpha_k|<R}\frac{1}{\alpha_k}\right|$ is bounded (α_k are the function roots).

Lemma 2. *If the function*

$$f(z)=Ce^{kz}\prod_{k=1}^{\infty}\left(1-\frac{z}{\alpha_k}\right)e^{\frac{z}{\alpha_k}} \tag{2}$$

satisfies the conditions (A) *and* (B), *then*

$$E=I(k)+\sum_{k=1}^{\infty}I\left(\frac{1}{\alpha_k}\right)\geqslant 0. \tag{3}$$

Proof. Take the function

$$\Phi(z)=\frac{f(z)}{f(z)}=e^{-2\pi iEz}\prod_{k=1}^{\infty}\left(\frac{1-\frac{z}{\alpha_k}}{1-\frac{z}{\alpha_k}}\right). \tag{4}$$

To evaluate $|\Phi(z)|$ we split up the infinite product into the factors

$$\Theta_1(z)=\prod_{k=1}^{\infty}\left(\frac{1-\frac{z}{\beta_k}}{1-\frac{z}{\beta_k}}\right);\quad \Theta_2(z)=\prod_{k=1}^{\infty}\left(\frac{1-\frac{z}{\gamma_k}}{1-\frac{z}{\gamma_k}}\right),$$

where β_k are the roots of our function satisfying the condition $\eta\leqslant\arg\beta_k\leqslant\pi-\eta$ $(\eta>0)$, γ_k are the remaining roots of $f(z)$.

Putting $\varphi=\arg z$ $(\eta<\varphi<\pi-\eta)$, we shall have

$$\ln|\Theta_2(z)|=\sum_{k=1}^{\infty}\ln\left|1+\frac{2iI\left(\frac{1}{\gamma_k}\right)}{\left(1-\frac{z}{\gamma_k}\right)}\right|<2|z|\sum_{k=1}^{\infty}\frac{-I\left(\frac{1}{\gamma_k}\right)}{\left|\frac{z}{\gamma_k}-1\right|}.$$

Noticing that $\min\limits_{(k)}\left|\frac{z}{\gamma_k}-1\right|\geqslant m=\min[\sin(\varphi-\eta),\ \sin(\varphi+\eta)]$, we obtain

$$\ln|\Theta_2(z)|<\frac{2\delta|z|}{1-\delta}\sum_{|\gamma_k|\leqslant\delta|z|}-I\left(\frac{1}{\gamma_k}\right)+\frac{2|z|}{m}\sum_{|\gamma_k|>\delta|z|}-I\left(\frac{1}{\gamma_k}\right),\quad (0<\delta<1).$$

Having an arbitrary ε, we choose a convenient δ and obtain

$$|\Theta_2(z)|<e^{\frac{\varepsilon}{3}z} \tag{5}$$

for $\eta<\arg z<\pi-\eta$, $|z|>R_\varepsilon$.

The function $\Theta_1(z)$ is the ratio of two functions of genus 0:

$$\vartheta(z)=\prod_{k=1}^{\infty}\left(1-\frac{z}{\beta_k}\right),\quad \bar{\vartheta}(z)=\prod_{k=1}^{\infty}\left(1-\frac{z}{\beta_k}\right),$$

because, in virtue of the inequality $\frac{1}{|\beta_k|}<-\frac{1}{\sin\eta}I\left(\frac{1}{\beta}\right)$, the series $\sum_{k=1}^{\infty}\frac{1}{|\beta_k|}$ is convergent. For the function of genus 0, we have

$$|\vartheta(z)|<e^{\frac{\varepsilon}{3}|z|}\quad(|z|>R_\varepsilon). \tag{6}$$

Furthermore (5), we have for $|z|>R_\varepsilon$,

$$|\vartheta(z)|>e^{-\frac{\varepsilon}{3}|z|} \tag{7}$$

except the points of some circles such that the sum of their radii does not exceed ζR, $(\zeta > 0)$, whenever their centres lie within the circle $|z| \leqslant R$.

The inequalities (5), (6) and (7) imply

$$\left|\frac{f(z)}{f(z)}\right| > e^{-2Ey-\varepsilon|z|} \quad (y = Iz) \tag{8}$$

for $\eta < \arg z < \pi - \eta$, $|z| > \max[R_\varepsilon, R'_\varepsilon, R_{\varepsilon,\zeta}]$ and z lying outside the exceptional circles. Let now the condition

$$\ln|f(iy_j)| > \left[h\left(-\frac{\pi}{2}\right) - \varepsilon\right] y$$

be fulfilled for a sequence of points $y_j \to +\infty$. It is obvious that for every y_j there exists a contour of diameter $\leqslant \zeta y_j$ that does not meet the exceptional circles and contains iy_j. By the minimum principle there is a point z_j on this contour such that

$$\ln|f(z_j)| > \left[h\left(-\frac{\pi}{2}\right) - \varepsilon\right] y_j \quad (|z_j - iy_j| < \zeta y_j) \tag{9}$$

$$\ln|f(z_j)| > \left[h\left(-\frac{\pi}{2}\right) - 2\varepsilon\right] |z_j| \tag{10}$$

for $\zeta < \dfrac{\varepsilon}{h\left(-\frac{\pi}{2}\right)}$. It is known ([6]) that $\ln|f(re^{i\varphi})| < [h(\varphi) + \varepsilon] r$ uniformly for $\left|\arg z - \dfrac{\pi}{z}\right| < \dfrac{\pi}{z} - \eta$. In virtue of the continuity of $h(\varphi)$ ([6])

$$\ln|f(z_j)| < \left[h\left(\frac{\pi}{z}\right) + 2\varepsilon\right] |z_j| \tag{11}$$

whenever $\zeta > 0$ is sufficiently small. From (10), (11) and (B) follows

$$\left|\frac{f(z_j)}{f(z_j)}\right| < e^{4\varepsilon|z_j|}. \tag{12}$$

Comparing (12) and (8), we obtain $-2E < 5\varepsilon(1+\zeta)$. Since $\varepsilon > 0$ is arbitrary $E \geqslant 0$ *.

Lemma 3. *If in the product*

$$\psi(z) = \prod_{-\infty}^{\infty} \frac{1 - \frac{z}{b_k}}{1 - \frac{z}{a_k}} \tag{13}$$

the numbers a_k and b_k alternate and are real, then

$I[\psi(z)] \geqslant 0$ *for* $Iz \geqslant 0$ *and* $b_{-1} < 0 < a_0$, $I[\psi(z)] \leqslant 0$ *for* $Iz \leqslant 0$ *and* $a_0 < 0 < b^0$.

Proof. For the sake of definitiveness suppose that $b_{-1} < 0 < a_0$. Then

$$\arg\psi(z) = \sum_{-\infty}^{\infty} [\arg(z - b_k) - \arg(z - a_k)],$$

and it is geometrically obvious that $0 < \arg\psi(z) < \pi$. We prove the theorem.

Necessity. Let

$$\Theta(z) = \frac{f(z)}{f(z)} = e^{2iEz} \prod_{k=1}^{\infty} \left(\frac{1 - \frac{z}{x_k}}{1 - \frac{z}{a_k}}\right). \tag{14}$$

According to Lemma 2, $E \geqslant 0$ and consequently $|\Theta(z)| < 1$ for $Iz > 0$.

---

* It follows from the lemma just proved that the class of functions satisfying the conditions (A) and (B) coincides with the class studied by M. Krein.

Mapping the unit circle onto the upper semi-plane by means of the function $F=\frac{1}{i}\frac{\Theta-1}{\Theta+1}$, we obtain

$$F(z)=\frac{1}{i}\frac{\Theta(z)-1}{\Theta(z)+1}=\frac{Q(z)}{P(z)}. \tag{15}$$

The function $F(z)$ is real.

$$I[F(z)]>0 \text{ for } Iz>0 \text{ and } I[F(z)]<0 \text{ for } Iz<0, \tag{16}$$

and all the roots of $P(z)$ and $Q(z)$ are real. Let us prove that they alternate.

It follows from (16) that the variation of $\arg F(z)$ around the circle with the centre on the real axis does not exceed 2π. Consequently, every circle containing two neighbouring roots of the function $Q(z)$ contains at least one root of the function $P(z)$, and every circle containing two neighbouring roots of $P(z)$ contains at least one root of $Q(z)$. Moreover, $F'(x)>0$, because $F(z)$ maps the upper semi-plane onto itself.

We now shall prove the necessity of the condition (D). The convergency of the series $\sum_{-\infty}^{\infty}\left(\frac{1}{k}-\frac{1}{b_k}\right)$ implies the representation of the function $F(z)$:

$$F(z)=\frac{B}{A}e^{Kz}\prod_{-\infty}^{\infty}\left(\frac{1-\frac{z}{b_k}}{1-\frac{z}{a_k}}\right)\quad\left[K=b-a+\sum_{-\infty}^{\infty}\left(\frac{1}{b_k}-\frac{1}{a_k}\right)\right]. \tag{17}$$

In virtue of Lemma 3 and (16), we have

$$-\pi<\arg\frac{B}{A}e^{Kz}<2\pi \text{ for } Iz>0,$$

that is possible only when $K=0$.

Sufficiency. If the conditions (C) and (D) are fulfilled, we have

$$F(z)=\frac{Q(z)}{P(z)}=\frac{A}{B}\prod_{-\infty}^{\infty}\left(\frac{1-\frac{z}{b_k}}{1-\frac{z}{a_k}}\right). \tag{18}$$

By Lemma 3 the function $F(z)$ maps the upper semi-plane either onto the lower semi-plane or onto itself. Since $F'(x)>0$ in at least one point,

$$IF(z)>0 \text{ for } Iz>0.$$

The function

$$\Theta(z)=\frac{F(z)-i}{F(z)+i}=\frac{f(z)}{\bar{f}(z)} \tag{19}$$

maps the upper semi-plane onto the unit circle and the lower semi-plane onto the exterior of the unit circle. Hence follow the conditions (A) and (B).

Received
20.XII.1942.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

MATHÉMATIQUES

SUR LES SOMMES PARTIELLES DES SÉRIES TRIGONOMÉTRIQUES

Par D. MENCHOFF

(Présenté par A. N. Kolmogoroff, de l'Académie, le 2. VII. 1943)

On sait qu'il existe des fonctions continues dont les séries de Fourier divergent en certains points (1). Toutefois on ne savait pas s'il existait une série de Fourier d'une fonction continue dont la suite de sommes partielles ne contient aucune sous-suite partout convergente. On peut démontrer qu'il existe des fonctions continues dont les séries de Fourier possèdent cette dernière propriété. En effet, on peut démontrer le

Théorème 1. *Il existe une fonction $f(x)$, continue pour toutes les valeurs de x et possédant une période 2π, dont la série de Fourier*

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx) \tag{1}$$

possède la propriété suivante: quelle que soit une suite croissante de nombres entiers et positifs λ_k, $k=1, 2, \ldots$, la suite de fonctions $S_{\lambda_k}(x)$, $k=1, 2, \ldots$, ne converge vers aucune limite finie au moins dans un point x, où

$$S_n(x)=\frac{a_0}{2}+\sum_{l=1}^{n}(a_l\cos lx+b_l\sin lx). \tag{2}$$

On peut démontrer de plus que l'ensemble de points de divergence de la série (1) considérée possède la puissance du continu. En effet, on peut démontrer le

Théorème 2. *Lorsque la série de Fourier* (1) *d'une fonction $f(x)$, partout continue et de période 2π, converge en chaque point, sauf peut-être aux points d'un ensemble fini ou dénombrable, on peut déterminer une suite croissante de nombres entiers et positifs ν_l, $l=1, 2, \ldots$, tels que*

$$\lim_{l\to\infty}S_{\nu_l}(x)=f(x) \tag{3}$$

pour toutes les valeurs de x, où $S_n(x)$ est définie par l'égalité (2).

L'ensemble de points de divergence d'une série trigonométrique étant mesurable B, il résulte du théorème 2 que la série de Fourier qui figure dans l'énoncé du théorème 1 diverge aux points d'un ensemble ayant la puissance du continu*.

* On sait que chaque ensemble mesurable B est fini, dénombrable ou bien contient un ensemble parfait (2,3).

Le théorème 2 est un cas particulier d'un théorème plus général dont l'énoncé est le suivant.

Théorème 3. *Supposons qu'une fonction $f(x)$ de période 2π soit sommable dans le segment $[-\pi, \pi]$ et continue en chaque point d'un segment $[a, b]$. Dans ces conditions, si la série de Fourier (1) de la fonction $f(x)$ converge partout dans le segment $[a, b]$, sauf peut-être aux points d'un ensemble fini ou dénombrable, il existe une suite croissante de nombres entiers et positifs ν_l, $l=1, 2, \ldots$, tels que la relation (3) soit vérifiée partout dans le segment $[a, b]$.*

On définit la fonction $f(x)$, figurant dans l'énoncé du théorème 1, comme une somme de deux fonctions continues $\psi_1(x)$ et $\psi_2(x)$ qui possèdent la propriété suivante: la suite de sommes partielles de la série de Fourier de chacune des fonctions $\psi_1(x)$ et $\psi_2(x)$ contient une sous-suite uniformément convergente.

On peut prouver que chaque fonction continue possède la même propriété, c'est-à-dire on a le

Théorème 4. *Quelle que soit une fonction $f(x)$, partout continue et de période 2π, on peut déterminer deux fonctions $\psi_1(x)$ et $\psi_2(x)$ qui possèdent les propriétés suivantes:*

a) $f(x)=\psi_1(x)+\psi_2(x) \qquad (-\infty<x<+\infty)$;

b) *les fonctions $\psi_1(x)$ et $\psi_2(x)$ sont partout continues et possèdent une période 2π;*

c) *les suites de sommes partielles $S_n^{(1)}(x)$ et $S_n^{(2)}(x)$, $n=1, 2, \ldots$, des séries de Fourier des fonctions $\psi_1(x)$ et $\psi_2(x)$ contiennent des sous-suites $S_{\lambda_k}^{(1)}(x)$ et $S_{\mu_k}^{(2)}(x)$, $k=1, 2, \ldots$, qui convergent uniformément dans l'intervalle infini $(-\infty, +\infty)$.*

Ce théorème est une conséquence immédiate d'un théorème plus général dont l'énoncé est le suivant.

Théorème 5. *Pour toute fonction $f(x)$ de période 2π, sommable dans le segment $[-\pi, \pi]$ et continue en chaque point d'un segment $[a, b]$ on peut définir deux fonctions $\psi_1(x)$ et $\psi_2(x)$ qui possèdent les propriétés suivantes:*

a) $f(x)=\psi_1(x)+\psi_2(x) \qquad (-\infty<x<+\infty)$;

b) *les fonctions $\psi_1(x)$ et $\psi_2(x)$ sont sommables dans $[-\pi, \pi]$ et possèdent une période 2π;*

c) *les suites de sommes partielles $S_n^{(1)}(x)$ et $S_n^{(2)}(x)$, $n=1, 2, \ldots$, des séries de Fourier des fonctions $\psi_1(x)$ et $\psi_2(x)$ contiennent des sous-suites $S_{\lambda_k}^{(1)}(x)$ et $S_{\mu_k}^{(2)}(x)$, $k=1, 2, \ldots$, qui convergent uniformément dans le segment $[a, b]$.*

L'idée de la démonstration du théorème 5 est la suivante. On définit des nombres entiers et positifs n_l, des nombres positifs ε_l et des polynômes trigonométriques

$$T_l(x)=\frac{\alpha_0^{(l)}}{2}+\sum_{n=1}^{\nu_l}(\alpha_n^{(l)}\cos nx+\beta_n^{(l)}\sin nx),$$

$l=1, 2, \ldots$, qui vérifient les conditions:

1) $n_s<n_{l_1} \quad \nu_s<n_l \quad (1\leqslant s<l-1,\ l=2, 3, \ldots)$;

2) $\varepsilon_l=\frac{1}{n_l 2^l} \qquad (l=2, 3, \ldots)$;

3) $\int\limits_{-\pi}^{\pi}|f(t)-T_l(t)|\,dt<\varepsilon_l \qquad (l=2, 3, \ldots)$;

4) $|f(x)-T_l(x)|<\varepsilon_l \qquad (a\leqslant x\leqslant b,\quad l=2, 3, \ldots)$,

et l'on démontre que les deux séries

$$T_1(x)+\sum_{j=1}^{\infty}[T_{2j+1}(x)-T_{2j}(x)] \quad \text{et} \quad \sum_{j=1}^{\infty}[T_{2j}(x)-T_{2j-1}(x)]$$

convergent presque partout dans l'intervalle infini $(-\infty, +\infty)$ respectivement vers des fonctions $\psi_1(x)$ et $\psi_2(x)$ qui possèdent toutes les propriétés indiquées dans l'énoncé du théorème 5.

Nous avons considéré jusqu'à présent les sommes partielles des séries de Fourier des fonctions continues. On pourrait aussi considérer les suites de sommes partielles des séries trigonométriques qui ne sont pas nécessairement des séries de Fourier. Il est bien connu que les coefficients d'une série de Fourier (1) d'une fonction continue tendent vers zéro avec $\frac{1}{n}$, c'est-à-dire

$$\lim_{n\to\infty} a_n = 0, \quad \lim_{n\to\infty} b_n = 0\ (^4). \tag{4}$$

On voit donc du théorème 1 qu'il existe une série trigonométrique à coefficients tendant vers zéro avec $\frac{1}{n}$ dont la suite de sommes partielles ne contient aucune sous-suite partout convergente. Toutefois on peut obtenir, pour les séries trigonométriques vérifiant la condition (4), un résultat plus précis. On peut, en effet, démontrer le

Théorème 6. *Il existe une série trigonométrique* (1) *à coefficients tendant vers zéro avec* $\frac{1}{n}$ *qui possède la propriété suivante: quelle que soit une suite croissante de nombres entiers et positifs* λ_k, $k=1, 2, \ldots$, *la suite des fonctions* $S_{\lambda_k}(x)$, $k=1, 2, \ldots$, *ne converge vers aucune limite finie presque partout dans* $[-\pi, \pi]$, *où* $S_n(x)$ *est défini par l'égalité* (2).

Le théorème 6 est, en même temps, une généralisation d'un résultat de M. Lusin (5) qui a donné l'exemple d'une série trigonométrique presque partout divergente dont les coefficients tendent vers zéro avec $\frac{1}{n}$.

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Moscou.

Manuscrit reçu
le 1. VII. 1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

MATHÉMATIQUES

REPÈRE ORTHOGONAL LIÉ INTRINSÈQUEMENT AVEC UNE CONGRUENCE ARBITRAIRE ET LES CONDITIONS POUR LA STRATIFICATION D'UN COUPLE DE CONGRUENCES

Par S. ROSSINSKI

(Présenté par A. N. Kolmogoroff, de l'Académie, le 15. XII. 1942)

§ 1. Soit M le milieu du rayon d'une congruence arbitraire non-dégénérée. Attachons à chaque rayon de cette congruence un repère triorthogonal de telle manière que son sommet coïncide avec le point M, que le vecteur unitaire i_1 de la première arête soit dirigé le long de la droite d'intersection du plan tangent à la surface moyenne de la congruence au point M et du plan passant par M et orthogonal au rayon de la congruence (c'est-à-dire avec le plan nommé moyen), et que le vecteur unitaire i_2 de la seconde arête soit aussi placé dans le plan moyen; donc, le vecteur unitaire i_3 de la troisième arête sera dirigé le long du rayon de la congruence*.

Il est évident que ces conditions caractérisent complètement le repère triorthogonal proposé qui est lié intrinsèquement avec une congruence arbitraire.

Soient

$$dM = \omega^\alpha i_\alpha; \quad di_k = \omega_k^\alpha i_\alpha, \qquad (k = 1, 2, 3)$$

les formules déterminant les déplacements infiniment petits du trièdre choisi; on a désigné ici par M le vecteur du point M.

Les formes de Pfaff ω^α, ω_k^α seront des formes différentielles linéaires de deux paramètres indépendants; quand on donne les valeurs de ces paramètres, on fixe dans l'espace la position d'un rayon de la congruence. Les vecteurs i_1, i_2, i_3 étant unitaires et mutuellement orthogonaux, on a pour les formes ω les conditions

$$\omega_l^l = 0, \quad \omega_l^k = -\omega_k^l \qquad (k, l = 1, 2, 3). \tag{1}$$

Les formes ω doivent d'ailleurs vérifier les équations de structure. Comme le repère choisi dépend de deux paramètres et quand ils sont donnés, sa position dans l'espace est fixée, les composantes ω^α et ω_k^α du déplacement instantané doivent être liées par $12 - 2 = 10$ relations linéaires, dont six sont déjà présentées par les égalités (1).

Supposons que les différentielles des deux paramètres indépendants sont présentées par les formes ω_1^3 et ω_2^3. Alors les quatre relations linéaires qui

* Si les plans indiqués coïncident pour chaque rayon, la surface moyenne sera minime et (i_3) sera la congruence de ses normales. Dans ce cas on peut diriger i_1, i_2 le long des lignes de courbure de la surface minime ou bien le long de ses lignes asymptotiques.

restent peuvent être écrites ainsi:

$$\omega^1 = \alpha\omega_1^3 + \beta\omega_2^3,$$
$$\omega^2 = \lambda\omega_1^3 + \mu\omega_2^3,$$
$$\omega^3 = \xi\omega_1^3 + \eta\omega_2^3,$$
$$\omega_1^2 = \rho\omega_1^3 + \sigma\omega_2^3;$$

d'ailleurs en vertu du choix spécial du repère il y aura certains liens entre les fonctions α, β, λ, ... , ρ, σ. En remarquant que le sommet du repère coïncide avec le milieu M du rayon, on trouve le premier lien:

$$\alpha + \mu = 0.$$

Ayant exigé que le vecteur i_1 soit dirigé le long de la ligne d'intersection du plan tangent en M à la surface moyenne et du plan passant par M et orthogonal au vecteur i_3, nous devons écrire l'égalité

$$\xi : \lambda = \eta : \mu. \qquad (2)$$

Ce sont les seules relations qui lient lès fonctions $\alpha, \beta, \lambda, \ldots, \rho, \sigma$. En désignant les rapports égaux (2) par γ, nous pouvons dire que *le système de Pfaff*

$$\left.\begin{array}{ll} \omega^1 = \alpha\omega_1^3 + \beta\omega_2^3, & \omega_l^l = 0 \\ \omega^2 = \lambda\omega_1^3 - \alpha\omega_2^3, & \\ \omega^3 = \gamma\omega^2, & \omega_k^l = -\omega_l^k \quad (k, l = 1, 2, 3) \\ \omega_1^2 = \rho\omega_1^3 + \sigma\omega_2^3, & \end{array}\right\} \qquad (3)$$

définit une congruence arbitraire de droites rapportée au repère choisi.

§ 2. Conditions générales de la stratification bilatérale d'un couple de congruences (j_1), (j_2) dont la congruence (3) des perpendiculaires communes est rapportée au repère $M(i_1, i_2, i_3)$.

Supposons qu'une congruence arbitraire (i_3) rapportée au repère choisi dans le § 1 est la congruence des perpendiculaires communes d'un certain couple de congruences (j_1), (j_2); nous désignons par j_1 et j_2 les vecteurs unitaires de leurs rayons dirigés le long des rayons correspondant à i_3. Soit P_k le point et aussi le vecteur de l'intersection des rayons j_k et i_3, C — le milieu (et le vecteur du milieu) du segment P_1P_2.

Posons

$$C = M + li_3, \quad P_1 = M + (l-a)i_3, \quad P_2 = M + (l+a)i_3.$$

Si θ est l'angle du rayon j_1 et du vecteur i_1, et φ — l'angle entre les rayons j_1 et j_2, on a

$$j_1 = i_1\cos\theta + i_2\sin\theta, \quad j_2 = i_1\cos(\theta+\varphi) + i_2\sin(\theta+\varphi).$$

En adoptant ces notations nous arrivons aux conditions générales suivantes pour la stratification bilatérale du couple (j_1), (j_2) dont (3) est la congruence

des perpendiculaires communes et qui est rapportée au repère $M(i_1, i_2, i_3)$:

$$
\begin{aligned}
&[d(l-a),\ \omega_1^3\cos\theta+\omega_2^3\sin\theta]+\frac{2a}{\sin\varphi}[d\theta,\ \omega_1^3\cos(\theta+\varphi)+\\
&+\omega_2^3\sin(\theta+\varphi)]+\Big\{\gamma(\lambda\sin\theta+\alpha\cos\theta)+\\
&+\frac{2a}{\sin\varphi}[\rho\sin(\theta+\varphi)-\sigma\cos(\theta+\varphi)]\Big\}[\omega_1^3\ \omega_2^3]=0,\\
&[d(l+a),\ \omega_2^3\sin(\theta+\varphi)+\omega_1^3\cos(\theta+\varphi)]+\frac{2a}{\sin\varphi}[d(\theta+\varphi),\ \omega_2^3\sin\theta+\\
&+\omega_1^3\cos\theta]+\Big\{\gamma[\alpha\cos(\theta+\varphi)+\lambda\sin(\theta+\varphi)]+\\
&+\frac{2a}{\sin\varphi}[-\sigma\cos\theta+\rho\sin\theta]\Big\}[\omega_1^3\omega_2^3]=0,\\
&[d(l+a),\ \{(\alpha+a-l)\sin(\theta+\varphi)-\lambda\cos(\theta+\varphi)\}\omega_1^3+\\
&+\{\beta\sin(\theta+\varphi)+(\alpha+l-a)\cos(\theta+\varphi)\}\omega_2^3]-\\
&-\frac{2a}{\sin\varphi}[d(\theta+\varphi)\{\lambda\cos\theta+(l+a-\alpha)\sin\theta\}\omega_1^3-\\
&-\{(l+a+\alpha)\cos\theta+\beta\sin\theta\}\omega_2^3]+\\
&+\Big\{\gamma\sin(\theta+\varphi)[\lambda\beta+\alpha(\alpha+a-l)]+\gamma\lambda\cos(\theta+\varphi)(l-a)+\\
&+\frac{2a}{\sin\varphi}[\{\sigma\lambda+\rho(l+a+\alpha)\}\cos\theta+\\
&+\{\rho\beta+\sigma(l+a-\alpha)\}\sin\theta]\Big\}[\omega_1^3\omega_2^3]=0,\\
&[d(l-a),\ \{(-\alpha-a-l)\cos\theta-\beta\sin\theta\}\omega_2^3+\\
&+\{\lambda\cos\theta+(-\alpha+l+a)\sin\theta\}\omega_1^3]-\\
&-\frac{2a}{\sin\varphi}[d\theta,\ \{\beta\sin(\theta+\varphi)+(l-a+\alpha)\cos(\theta+\varphi)\}\omega_2^3-\\
&-\{(l-a-\alpha)\sin(\theta+\varphi)+\lambda\cos(\theta+\varphi)\}\omega_1^3]+\\
&+\Big\{-\gamma\lambda(l+a)\cos\theta-\gamma[\lambda\beta+\alpha(\alpha-l-a)]\sin\theta-\\
&-\frac{2a}{\sin\varphi}[\{\rho\beta+\sigma(l-a-\alpha)\}\sin(\theta+\varphi)+\\
&+\{\sigma\lambda+\rho(l-a+\alpha)\}\cos(\theta+\varphi)]\Big\}[\omega_1^3\omega_2^3]=0,\\
&2l\cos\varphi+2a\cos(2\theta+\varphi)+(\beta+\lambda)\sin(2\theta+\varphi)=0,\\
&[da,\ dl+\gamma(\lambda\omega_1^3-\alpha\omega_2^3)]+\frac{2a^2}{\sin^2\varphi}[d\varphi,\ d\theta+\rho\omega_1^3+\sigma\omega_2^3]=0.
\end{aligned}
\tag{4}
$$

Ces conditions peuvent être appliquées à l'étude d'un couple arbitraire de congruences bilatéralement stratifiable.

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Manuscrit reçu
le 27. VI. 1942.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

PHYSICS

«TRUNK» BEAMS OF THE AUGER SHOWERS AS OBSERVED BY THE METHOD OF COINCIDENCES

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In the previous notes ([1,2]) we have considered the space distribution of particles within the Auger showers.

The extremely abrupt $\left(\sim \frac{1}{r}\right)$ increase of the density of the current when approaching the shower axis is a remarkable peculiarity of this distribution. The presence in the centre of the shower of condensations—of «trunks» characterized by exceedingly high intensity of the current—was taken notice of by Auger in his first experiments.

Simple considerations show that the average energy of particle when approaching the axis, increases, speaking roughly, also as $\frac{1}{r}$. If the critical energy corresponds to the average radius (of the order of 100 m), then at a distance of one metre from the axis, an energy of order 10^{10} eV ([1]) is to be expected. As for the density of the «average» showers at an elevation as high as the altitude of mt. Jungfrau at the above distance from the axis, it is of the order of many hundreds of particles per square metre.

The investigation of similar beams of particles having such high energies undoubtedly offers considerable interest.

It is easy to see how the usual method of observation of coincidence in two or three counters ought to be changed to register coincidences that are due solely to the cases when the axis of the shower passes close to the centre of the arrangement of the counters. It is obvious that such showers would be distinguished by groups of several counters having their axes at a small distance from one another, if the number (n) of the elements in the scheme of coincidences is sufficiently large.

Because of the existence of an extremely dense trunk, showers with an r smaller than a certain r_{max} are registered with a probability close to unity, while this probability when $r > r_{max}$ decreases with the increase of r at an extremely high rate (the greater the number of the elements of the coincidence scheme, the greater the rate).

Under these conditions, r_{max} depends but little on the number of counters registering the showers. The greater this number; the greater is the abruptness with which the definite limit «radius of action» of the showers registered by the scheme is obtained. The magnitude of this radius is determined by the area of the axial cross-section of the counters (this radius being proportional to the area of the cross-section).

All these characteristic features are derived from the following calculations. To begin with we make the assumption that the fluctuations of the number of particles per area unit of the cross-section of the shower obey Poisson's law.

Inasmuch as the peripheral regions of the shower are of no significance for the effects considered, we adopt for the distribution of the average density the law $\rho = \frac{\rho_0}{r}$ (when $r < R$), and $\rho = 0$ (when $r > R$), R being the radius of the shower.

The probability of a coincidence is determined by the parameter $r_0 = \frac{N\sigma}{2\pi R}$ [see preceding note (1)]. N is the total number of particles in the shower; R is the «half» radius of the shower, and σ is the area of a counter.

It is well to point out here that the formula (5) in my paper (1) may be represented in the form

$$C_2 = 2\pi r_0^2 \Phi \int_{\left(\frac{D}{R}\right)}^{\infty} \frac{e^{-t}dt}{\sqrt{t^2 - \left(\frac{D}{R}\right)^2}},$$

where C_2 is the number of double coincidences per unit time; Φ is the number of showers (with an energy corresponding to a given r_0 per unit surface area and unit time; D is the distance between the axes of the counters).

In the case under consideration $D < r_0$, which circumstance is essential for the subsequent. For counters of usual size this condition may be justified practically for the entire region of the spectrum of the primary particles causing Auger showers. Besides, in the case when the showers are observed at a considerable altitude [as high, for instance, as the Jungfrau saddle (3)], the size of the counters may be chosen to justify the condition

$$r_0 \ll R$$

over the whole of the region just mentioned. At sea level this condition is not justified.

The number of coincidences in n counters caused by the showers from the primary particles of a definite energy may be put equal to

$$C_n = \Phi \int_0^R \left(1 - e^{-\frac{r_0}{r}}\right)^n 2\pi r\, dr. \qquad (1)$$

Remember that r_0 is that distance from the shower axis, at which the number of particles crossing a single counter is on the average equal to unity. $\frac{r_0}{r}$ is the average number of particles passing across the counter at a distance equal to r.

Integrating by parts gives:

$$C_n = 2\pi r_0^2 \frac{1}{2} \left\{ \left(1 - ne^{-\frac{r_0}{R}} + \frac{n(n-1)}{1\cdot 2} e^{-\frac{2r_0}{R}} - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} e^{-\frac{3r_0}{R}} + \dots \right.\right.$$

$$\left. \dots + (-1)^n e^{-n\frac{r_0}{R}} \right) \left(\frac{R}{r_0}\right)^2 + \left(n e^{-\frac{r_0}{R}} - \frac{n(n-1)}{1\cdot 2} 2e^{-2\frac{r_0}{R}} + \right.$$

$$+ \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} 3e^{-\frac{3r_0}{R}} - \dots - (-1)^{n+1} ne^{-\frac{nr_0}{R}} \Big) \frac{R}{r_0} - n \int_{\frac{r_0}{R}}^{\infty} \frac{e^{-x}}{x} dx +$$

$$\left. + \frac{n(n-1)}{1\cdot 2} 2^2 \int_{\frac{2r_0}{R}}^{\infty} \frac{e^{-x}dx}{x} - \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3} 3^2 \int_{\frac{3r_0}{R}}^{\infty} \frac{e^{-x}dx}{x} \dots (-1)^n n^2 \int_{\frac{nr_0}{R}}^{\infty} \frac{e^{-x}}{x} dx \right\}. \qquad (2)$$

In the sequel the factor Φ is omitted.

It may be said beforehand that with $R \gg r_0$ the result of the integration should not practically depend on the value of R. That it is actually so, appears from a closer investigation of equation (2).

Indeed, in the limit case $\left(\frac{R}{r_0} \gg 1\right)$ the exponential terms in the first line of equation (2) may be neglected* (with $n \geqslant 3$), while for the exponential integrals of equation (2) there holds the approximate expression:

$$\int_{\frac{mr_0}{R}}^{\infty} \frac{e^{-x}}{x}\, dx = -\ln \gamma \frac{mr}{R}, \text{ where } \gamma = e^c = 1.78 \ (c \text{ is Euler's constant}).$$

It may further be seen that when $n \geqslant 3$, the sum of the coefficients of these integrals is equal to zero. Therefore the equation (2), after the substitution of integrals with logarithms, is reduced to the following expression:

$$C_n = -2\pi r_0^2 \frac{1}{2}\left(2^2 \frac{n(n-1)}{1 \cdot 2} \ln 2 - 3^2 \times \right.$$
$$\times \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \ln 3 + \ldots$$
$$\left. \ldots + (-1)^n n^2 \ln n\right). \qquad (3)$$

Thus, we get the following simple result:

$$C_n = 2\pi r_0^2 K_n, \qquad (4)$$

where K_n is a certain numerical factor, depending only upon the number of elements in the scheme of coincidences. R does not enter into equation (4) explicitly.

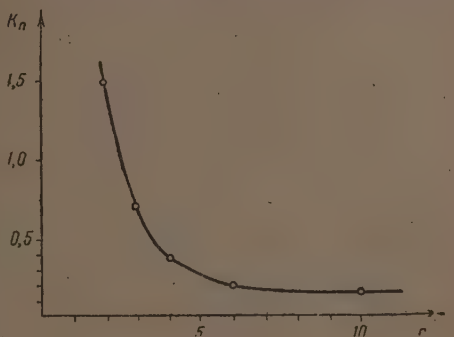

Fig. 1

The values of K_n are given in the table and in the graph ** (Fig. 1).

The area of the cross-section of a counter (σ) enters the formula only through the value of r_0. At any number of n, the probability of coincidence is proportional to the square of this area. The integral (1) with r as its upper limit gives the number of coincidences caused by the showers, whose axis passes at a distance smaller than r from the centre of the arrangement of the counters. This number

n	3	4	6	10
K_n	0.73	0.37	0.18	0.11

of coincidences, consequently, is given again by equation (2), if R in it is substituted with r.

Fig. 2 shows the curves $C = f(r)$ calculated thus, and the curves $\frac{dC}{dr}$ for different values of n obtained from them by means of graphic differentiation. The initial portions of the curves $C = f(r)$ are represented by parabolas ($C = \pi r^2$).

* The sum of these terms is of order $\left(\frac{r_0}{R}\right)^{n-2}$ when $\frac{r_0}{R} \ll 1$.

** The curve of the graph in Fig. 1 decreases very slowly. Not so with the small «parasitary» showers, arising in the vicinity of the counters: the probability of their registration falls abruptly with the increase of n. This makes it certain that with sufficiently large n, the background of parasitary coincidences may be neglected.

As the graph indicates; on observing, for example; sixfold coincidences (or with $n>6$), the action of the showers is practically sharply confined within the radius r_0. Within certain limits, r_0 is approximately proportional to the energy of the primary particle.

For counters with a cross-section area equal to $\sigma=100$ cm^2 we have $r_0=4.5$ m, corresponding to the average effective energy of the primary particles ($E\cong 10^{14.5}$ eV) on the Jungfrau altitude.

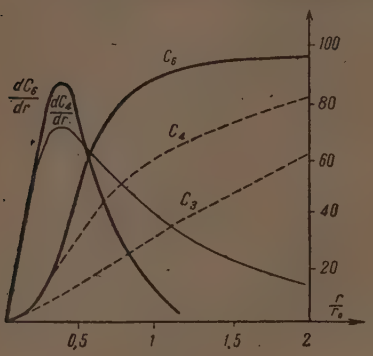

Fig. 2

The absolute intensity and the spectral distribution of the primary radiation being known, we can calculate the absolute number of coincidences per unit time.

This number of coincidences is determined by the formulae which we have already made use of in our previous paper ([2]) [the equation (7) given below is the formula (2) of ([2]) with certain corrections taken into account].

Supposing the average energy of particles forming the shower to be

$$\varepsilon=\frac{10^{10}}{r}\ \text{eV}$$

(where r is the distance from the axis of the shower expressed in metres), and taking according to the graph (Fig. 2) that the average distance r for the shower generated by the primary particle of a definite energy is of order $0.5r_0$ (sixfold coincidences), we can calculate from formula (7) the average value of $\left(\frac{1}{r}\right)$; and; consequently, the value $\bar{\varepsilon}$ as an average over the entire spectrum of the primary particles ([2]):

$$\left(\frac{1}{r}\right)=0.5\,\frac{2\pi R}{\sigma}\left(\frac{1}{N}\right)=0.5\,\frac{2\pi R}{\sigma}\,\frac{\int_{y-3}^{y=5.3}\frac{1}{N}N^2f(y)\,dy}{\int_{3}^{5.3}N^2f\,dy},$$

$$\ln(Nf)=-\beta_2y^2-\delta_2=-0.185y^2-43.9+\ln A;$$

$$\ln(N^2f)=\alpha_1y-\beta_1y^2-\delta_1+\ln A,$$

$$\left(\frac{1}{r}\right)=0.5\,\frac{2\pi R}{\sigma}\,\frac{e^{-\delta_2}\int_{3}^{5.3}e^{-\beta_2y^2}\,dy}{e^{-\frac{\alpha_1^2}{4\beta_1}-\delta_1}\int_{3-\frac{\alpha_1}{2\beta_1}}^{5.3-\frac{\alpha_1}{2\beta_1}}e^{-\beta_1y^2}\,dy}=$$

$$=0.5\,\frac{2\pi\cdot 7\cdot 10^3}{10^2}\,\frac{e^{-43.9}\int_{3}^{5.3}e^{-0.185y^2}dy}{e^{-34.8}\int_{-2.6}^{-0.3}e^{-0.37y^2}dy}=10^{-2.15}\text{cm}^{-1}. \qquad (5)$$

The lower limit of the integral ($E=10^{13}$eV) is determined by the condition $r_0=D$. The upper limit corresponds to the cascade maximum. The nu-

merical values refer to the Jungfrau altitude (3400 m, $t = 17.2$) and the counters having a cross-section area of $\sigma_2 = 100$ cm^2. Thus,

$$\frac{1}{\left(\frac{1}{r}\right)} \cong 1 \text{ m} \quad \text{and} \quad \bar{\varepsilon} \cong 10^{10} \text{eV}.$$

The expected number of coincidences per hour may be determined by using the Auger data for double coincidences when $D = 4$ m.

For the Auger counters ($\sigma_1 = 200$ cm^2) we have the experimental value

$$C_2 (D = 4\text{m}) = 60 \text{ coincidences per hour.}$$

According to the calculation:

$$C_2 (D = 4\text{m}) = 0.9 \cdot 2\pi \bar{r}_0^2 \Phi^*$$

(where $\bar{r}_0^2$ is the value of r_0^2 averaged over the spectrum of the primary particles for the Auger counters). The number of sixfold coincidences we are interested in, according to the calculation, equals

$$C_6 (D = 0) = 0.18 \cdot 2\pi \bar{r}_0^2 \Phi \left(\frac{\sigma_2}{\sigma_1}\right)^2.$$

Hence in our case $\left(\frac{\sigma_2}{\sigma_1} = \frac{1}{2}\right)$

$$C_6 (D = 0) = \frac{0.18}{0.9} \cdot \frac{1}{4} \cdot 60 = 3 \text{ coincidences per hour.} \qquad (6)$$

In the case of fourfold coincidences the number of recorded coincidences is twice as large.

The results obtained show that along with the ionization method (the observation of pulses), the schemes of coincidences also give definite possibilities for observing the central trunks of the atmospheric showers. There is no doubt that the method of coincidences in certain respects may be essentially more efficient than the use of ionization chambers.

Moreover, we wish to add the following remarks.

1. The calculation of the probability of double coincidences in the case of a considerable distance D between the counters has been performed earlier by applying the method of approximations, which, as may be seen, gives an exaggerated value. In the limit case ($D = 0$) we may perform the calculation by the method of approximation as well as exactly. Comparison of the results gives a certain correction coefficient equal to 0.6, by which the results received earlier are to be multiplied [equation (2) of note (2)]. On the other hand, however, the former expression must also be corrected by introducing the factor ln 10 which we have erroneously omitted.

With these corrections introduced and after setting right some misprints, we get:

$$C_2 (D = 4\text{m}) = \frac{1}{2\pi} \frac{\sigma^2}{R^2} \alpha \int N^2 f \, dy \int_{\frac{D}{R}}^{\infty} \frac{e^{-t} \, dt}{\sqrt{t^2 - \left(\frac{D}{R}\right)^2}} =$$

$$= \frac{\sigma^2}{2\pi R^2} \cdot 0.5 \cdot 2.3 \cdot 0.6 \cdot 1.2 \cdot 10^{20} \cdot 1.16 e^{-34.9} \cdot 3.03 = \frac{\sigma^2}{2\pi R^2} \cdot 2.92 \cdot 10^{4.85}. \qquad (7)$$

* The numerical coefficient $0.9 = 0.6 \cdot 1.5$, where 1.5 is the corresponding coefficient of formula (5) of our first note, while 0.6 is the coefficient of correction (see below).

The ratio $\frac{C_{exp}}{C_{calc}}$ undergoes no essential alteration, while the ratio $\frac{\nu_{exp}}{\nu_{calc}}$ (ionization pulses) must be decreased 2.3 times.

As a result, $\frac{C_{exp}}{C_{calc}}$ and $\frac{\nu_{exp}}{\nu_{calc}}$ turn out to be about the same (of order 6 at sea level), and the agreement pointed out earlier between the data secured by means of counters and by means of the ionization chambers becomes closer still.

2. According to equation (1), in the limit case $\frac{D}{R} \ll 1$ the number of coincidences in the group of counters practically does not depend on D. At small distances, the number of coincidences increases only slightly* with the decrease of distance between the counters (roughly estimated, within the limits of $D = 0 \div 4$ m altogether 1.5÷2 times). But, on the other hand, the data of Auger and collaborators relating to coincidences in two counters show an apparently abrupt increase at small distances. As may be easily seen after a simple calculation, this increase is undoubtedly caused by the coincidence due to single particles passing in directions close to the horizontal. If three counters are used for observation (two of them having centres on the same vertical), it is necessary to take into account that in the case of considerable proximity of the counters, there appear additional possibilities of coincidences due to two particles passing through various combinations of counters,—the effective solid angle of the recording apparatus increases.

Unpublished observations made on mt. Elbruz by a board of workers of this Institute testify to the fact that in the case of small distances between the counters, the probability of registering showers depends very little on that distance—a peculiarity characteristic of the Auger showers.

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Received
5. VI. 1943.

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* This increase is solely due to the softest components of the primary spectrum ($E = 10^{12} \div 10^{13}$ eV).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

PHYSICS

THE ELEMENTARY SKIN-EFFECT

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Phenomena that take place in the surface layers of an infinitely long metal cylinder, or of a plate of infinite extension, under a strongly developed skin-effect are not different from those observed in the surface layer of a metallic half-space when outside of it an alternating electric or magnetic field directed parallel to its surface is in action. In this case we have the skin-effect in its most simple form and call it an elementary skin-effect. It may be interpreted as a penetration of the electromagnetic field from the outward medium into the conducting half-space.

One may distinguish the cases of a periodic and aperiodic alternating field. The first case has been thoroughly investigated ([1], [2]). An aperiodical field we shall consider in the form of an instantaneous constant electric field of intensity X_0 or magnetic field M_0.

In the surface layers of the half-space the field X_0 gives rise to an electric current of linear density g_1 which we describe as an overlay of current. The field M_0 excites in the surface layers a magnetic flux of induction. Its linear density φ is described as an overlay of flux.

By definition

$$g=\int_0^\infty \gamma X\, dz \quad \text{and} \quad \varphi=\int_0^\infty \mu M\, dz,$$

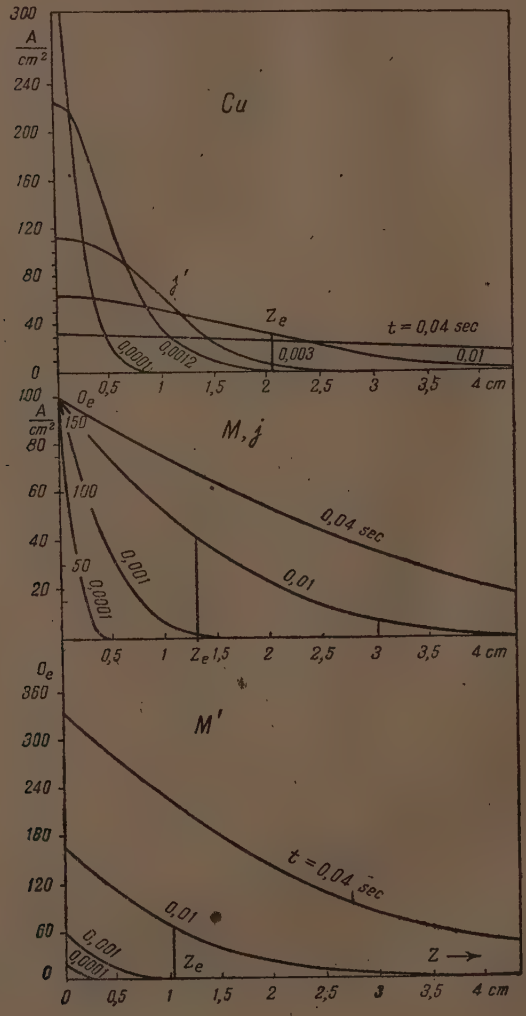
where z is an inwardly directed normal to the surface, and X or M are the values of the field within the half-space. The appearance of the overlays gives rise to secondary fields M' and X', perpendicular to them, and, correspondingly, to secondary overlays φ' and g'.

The overlay of current g' acts upon the surface of the half-space as an infinite plane current, and gives rise to a perpendicular magnetic field $2\pi g'$ on the one side and to a corresponding magnetic field $-2\pi g'$ on the other. Owing to this, the field is zero at a sufficient depth, whereas outside the half-space it is equal to $4\pi g'$. The same is true of the overlay φ'. The overlays g' and φ' are related to the primary fields M_0 and X_0 by the equations

$$M_0=4\pi g' \text{ and } X_0=\frac{d\varphi'}{dt}. \qquad (1)$$

In practice the strengthening of the magnetic field at the surface of a conductor is brought about when the reverse lines of the induced current do not weaken greatly the magnetic field at the surface of the body, for instance, at the equator of a copper sphere in a high frequency field. Here the sphere plays the rôle of a screen with $\mu=0$ pushing away the lines of the current.

Since a changing field is obtained also when the body and the field are in motion with respect to each other, it is possible to strengthen a constant magnetic field, too. Such a strengthening can be accomplished by means of special compressors of the magnetic flux. The velocities and the stores of



energy; necessary for the creation of short-lived magnetic fields of high intensity, can be obtained in different ways (from a copper-wing of a propeller, a discharge of a strongly cooled copper projectile, etc.). By the use of copper obstacles, of counter-motion and of a piston one may intensify the field many times.

1. If there is no magnetic hysteresis and viscosity which greatly complicate the process ([3-5]), the fields X and M at a constant magnetic permeabi-

lity μ at depth z and at a moment t after the appearance of X_0 and M_0 have the following values (1)

$$X = X_0 F(u) \text{ and } M = M_0 F(u).$$

Here $F(u)$ is Gauss' integral:

$$F(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-u^2} du, \quad u = z \sqrt{\frac{\pi\mu\gamma}{t}}.$$

The secondary fields are expressed thus:

$$M' = Q \sqrt{t}\ \Phi(u) X_0, \quad X' = \frac{f(u)}{Q\sqrt{t}} M_0, \tag{2}$$

where

$$Q = 4 \sqrt{\frac{\pi\gamma}{\mu}}, \quad f(u) = \frac{2}{\sqrt{\pi}} e^{-u^2} \text{ and } \Phi(u) = \frac{f(u)}{2} - uF(u).$$

The distribution of the fields X, M, X' and M' inside the half-space is shown in the figure; the electric field X is represented by the current density $j = \gamma X$. The curves refer to the time ranging from $t = 0.0001$ to $t = 0.04$ sec., as from the beginning of the process. They have been built for copper with $\gamma = 5.8 \cdot 10^{-4}$ CGSm and $\mu = 1$.

It may be seen from the graph that the depth of penetration z_e is greater in the case of eddy currents (the curves j'), than in the case of the current j and field M. This conclusion may turn out to be of practical value for electromagnetic defectoscopy by direct current.

The formulae for primary overlays are

$$g = \frac{X_0}{\pi} \sqrt{\frac{\gamma t}{\mu}}, \quad \varphi = \frac{M_0}{\pi} \sqrt{\frac{\mu t}{\gamma}}.$$

The work expended in the formation of an overlay is calculated by means of Poynting's theorem:

$$W_M = \frac{M_0^2}{4\pi} \sqrt{\frac{\mu t}{\gamma}} = \varphi g'; \quad W_X = \frac{2X_0^2}{3\pi} \sqrt{\frac{\gamma t^3}{\mu}} = \frac{2}{3} g\varphi'.$$

2. Suppose the half-space to represent a ferromagnetic crystal with a residual induction B_J parallel to the plane $z = 0$. Under the influence of the field X_0 there arises the field M'. Until it remains below the coercive force of the crystal H_c, the permeability is $\mu = 1$, and, according to (2), we have

$$M_0' = 4X_0 \sqrt{\gamma t}.$$

The field M_0' attains the value H_c at the moment

$$t_c = \frac{1}{16\gamma} \left(\frac{H_c}{X_0} \right)^2.$$

For $t = t_c$ the overlays are expressed as

$$g_c = \frac{H_c}{4\pi}, \quad \varphi_c' = X_0 t_c, \quad W_c = \frac{2}{3} g_c \varphi_c'.$$

If M' is antiparallel to B_J, then for $z = 0$, when $t > t_c$, a leap-like change of magnetization sets in. There appears the overlay

$$\varphi' = 2B_J z = J_c,$$

where J_e is the impulse of the electric field. With X_0 constant the impulse

$$J_e = X_0 (t - t_c).$$

For $t > t_c$ the density of the current j and induction $2B_\perp$ are moving with a steep front at a constant rate $\frac{X_0}{2B_\perp}$.

The work expended in the formation of the overlay is

$$W_{fh} = \frac{2}{3} g_c \varphi_c + g\varphi'.$$

When a field $M_0 > H_c$, antiparallel to $B_\perp$, appears outside a crystalline ferromagnetic, the induction changes to $2B_\perp$ under the action of a field

$$\Delta M = M_0 - H_c.$$

The overlay $g' = \gamma X' z$, where $z = z_e$. As

$$4\pi g' = \Delta M \quad \text{and} \quad X' = 2B_\perp \frac{dz}{dt},$$

we have

$$2\pi\gamma \cdot 2B_\perp z^2 = J_m,$$

where J_m is the impulse of the magnetic field, which for ΔM constant is equal to $\Delta M t$.

There arises an overlay of magnetic flux

$$\varphi = 2B_\perp z = \sqrt{\frac{2B_\perp J_m}{2\pi\gamma}}.$$

The induction $2B_\perp$ and j' move to the inside of the medium with a steep front at a decreasing velocity.

The Joule heat involved in the change of magnetization of 1 cm² of layer is

$$W_J = \frac{\Delta M 2B_\perp z}{4\pi} = \varphi g'.$$

The work including that absorbed in hysteresis

$$W_h = \frac{H_c 2B_\perp z}{4\pi} = \frac{H_c \varphi}{4\pi}$$

is

$$W_{fh} = \frac{M_0 \varphi}{4\pi}.$$

Similar phenomena take place upon the magnetization of plates and cylinders. They have been discussed earlier ([1],[6]). The formulae established there are useful in the calculation of the dynamic magnetization of iron, in particular, in the defectoscopy of rails and in the design of electromagnetic brakes.

Received
29.V.1943.

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1943. Volume XLI, № 2

CHEMISTRY

CATALYTIC DEHYDROGENATION OF *p*-CYMENE

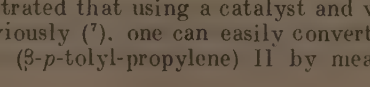
By A. A. BALANDIN, Corresponding Member of the Academy, G. M. MARUKIAN and R. G. SEIMOVICH

p-Cymene, a hydrocarbon that is closely related to terpenes, is capable of transformations common to hydrocarbons of the aromatic series. Owing to the inertness of its isopropyl group, the latter ordinarily does not participate in reactions (including the introduction of cymene into the living organism). This group is attacked in some oxidation reactions, among which should be mentioned the formation of cumic, *p*-(α-hydroxy-isopropyl) benzoic, and partly of *p*-isopropenyl-benzoic acids under the action of oxygen, water and light (1), the formation of *p*-(α-hydroxy-isopropyl)-benzoic acid under the action of potassium permanganate (2), the formation of *p*-toluic acid (3) and of methyl-*p*-tolyl-ketone (2) when oxidized with nitric acid, as well as of methyl-*p*-tolyl-ketone and of *p*-methyl-hydrotropaldehyde $CH_3.C_6H_4.CH(CH_3).CHO$(4) under the action of chromyl chloride.

Instead of such transformations; however; the present authors have considered the possibility of a direct conversion of the isopropyl group into the isopropenyl one without the other parts of the molecules being involved. Such a reaction has not been described as yet in the literature. In a patent(5) cymene was mentioned in a list of cyclic hydrocarbons yielding substituted styrenes when the vapours of the former are passed at 450—700° over Ce_2O_3 and ZnO; promoted by oxides of W; Mo and U.

However; the yields were not mentioned and it remains unknown whether the cleavage of the carbon skeleton takes place or not; as there exists another patent claim (6), according to which pyrolysis of cymene yields no less than 50 per cent of *p*-methyl-styrene, so that this reaction has been proposed as a method for the production of the latter. The exact temperature was not mentioned either.

We have demonstrated that using a catalyst and working according to the method described previously (7). one can easily convert cymene I into 1-methyl-4-isopropenyl-benzene (β-*p*-tolyl-propylene) II by means of catalytic dehydrogenation:



It should be noted that up to the present 1-methyl-4-isopropenyl-benzene could only be synthetized by comparatively cumbersome procedures, such as by the action of methyl-magnesium-iodide upon methyl-*p*-tolyl-ketone (8) or upon ethyl-*p*-toluate (9), as well as by the pyrolytic destruction of β-*p*-tolyl-crotonic (10) and of β-hydroxy-*p*-tolyl-butyric (11) acids.

Experimental. *p*-Cymene; b. p. 173.5—179° (the main part passed over at 175°), $d_4^{20} = 0.8639$, $n_D^{20} = 1.4935$ [data from literature: b. p. 175—176°, $d_4^{15} = 0.8631$, $n_D^{15} = 1.49474$ (12)], was passed over a solid catalyst under conditions that had been found to be an optimum for the dehydrogenation of ethyl-benzene into styrene (6), i. e. at 625° and a constant space velocity of 500—900 cm³ of the liquid per 1 l of the catalyst per hour. The condensate was collected and weighed; in a separate sample the bromine value was determined and the main amount was distilled using a fractionating column of the Fenske type, having an efficiency of 25 theoretical plates; packed with copper rings. Experiments were carried out both with and without dilution of cymene with carbon dioxide. The results obtained are tabulated below (Table 1).

Table 1

Dehydrogenation of Cymene

Nos. of runs	*t* in °C	Duration of work of catalyst, hrs	Space velocity, cm³/l/hr	Dilution	Yield of condensate in %	Concentration of methyl-isopropenyl-benzene in the condensate, weight %	Yield of methyl-isopropenyl-benzene based on cymene passed in %
53	625	55	615	without CO_2	88.3	58	52.3
54	625	56	880	» »	85	57.9	50
55	625	56.5	900	with CO_2	84	62.8	51.5
60	625	76.0	600	» »	93	64.1	63.4
61	630	81.0	570	» »	93.0	61.2	57.7
62	625	81.5	500	» »	—	23.8	—

It may be seen that dehydrogenation proceeds intensely, the dehydrogenation percentage being rather high and amounting at 625° to 63 per cent based on cymene passed, the condensate containing up to 64 per cent of unsaturates in the presence of a diluent; without a diluent under the same conditions the yield is up to 52 per cent of cymene passed, the concentration of 1-methyl-4-isopropenyl-benzene being 58 per cent.

In run No. 60 4.960 l of gas (without CO_2) were collected, containing only 7.2 per cent of unsaturates. The distillation of the condensate gave a fraction, b. p. 186—187°, containing 92.7 per cent of 1-methyl-4-isopropenyl-benzene (according to the bromine value), $d_4^{20} = 0.8996$, $n_D^{20} = 1.5329$ [data available in literature: b. p. 184—185°, $d_4^{18.7} = 0.9022$; $n_D^{18.7} = 1.53447$ (13)]. Like styrene the 1-methyl-4-isopropenyl-benzene obtained undergoes polymerization when heated or under the action of concentrated sulphuric acid in the cold. Benzoyl peroxide promotes polymerization. The catalyst can be easily regenerated by blowing air through it.

Table 1 shows that the catalyst having worked for 81 hrs without regeneration yields 57.7 per cent of methyl-isopropenyl-benzene, based on cymene passed. Run No. 62 was carried out with ethyl-benzene. In Table 2 comparative data on the dehydrogenation of ethyl-benzene and cymene are reported. The experiments were carried out over a fatigued catalyst which had been in use during 30 hrs before these experiments were started. The catalyst has not been regenerated.

Comparing the runs with ethyl-benzene and those with cymene (Table 2) to each other, it may be seen that the dehydrogenation of cymene proceeds much easier than that of ethyl-benzene under similar conditions.

Table 2

Comparative Data on the Dehydrogenation of Cymene and Ethyl-benzene

Nos. of runs	t in °C	Duration of work of catalyst, hrs	Substance passed	Dilution	Concentration of styrene or methyl-isopropenyl-benzene in the condensate, weight %
2	600	30.0	ethyl-benzene	without CO_2	26
3	600	34.0	cymene	» »	44.5
4	620	37.0	cymene	» »	52.2
12	600	43	ethyl-benzene	with CO_2	27.9
19	610	47.5	cymene	without CO_2	54.2

Thus, the introduction of two methyl groups in the molecule of ethyl-benzene, one into the nucleus, and the other into the side-chain in the α-position, favours dehydrogenation.

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Received
9. VII. 1943.

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1943. Volume XLI, № 2

CHEMISTRY

BEHAVIOUR OF AMINOHYDROXY ACIDS UNDER CONDITIONS OF ACID HYDROLYSIS OF PROTEIN

By M. M. BOTVINIK and E. A. MOROZOVA

(Communicated by S. S. Nametkin, Member of the Academy, 22. VI. 1943)

Within recent years substantial progress can be recorded in the study of aminohydroxy acids. Their preparation is no longer a matter of considerable difficulty after the method of West and Carter has been published (1).

In 1941 a simple and elegant method was developed for total determination of aminohydroxy acids by oxidizing them with natrium periodate (2), and there is every reason to expect now that the gap in our conceptions about the quantitative content of these acids in proteins will be filled in the near future. As for stability of aminohydroxy acids upon heating with mineral acids, it still remains an open question. According to many authors, considerable decomposition of aminohydroxy acids is observed in the process of hydrolysis of proteins. Thus, Knoop and associates assert that aminohydroxy acids show a poor stability when heated with mineral acids. This conclusion is upheld by Dakin (4) and others (11-12). The results of their analyses are referred to in Tables 1 and 2.

From these tables it follows that aminohydroxy acids are deamined to a great measure on the hydrolysis of protein. In the light of the work carried out on the estimation of aminohydroxy acids in proteins and from their physiological significance, that question appears to be very essential. If aminohydroxy acids are decomposed in the very process of hydrolysis, then the figures showing their content in the hydrolysate mixture may be lower than they actually are. This question calls therefore for special inquiry.

From comparing literary data on the decomposition of aminohydroxy acids under the action of mineral acids it may be inferred that no due attention was paid by the authors to the temperature regime. Meanwhile temperature may be expected, by analogy with the behaviour of other amino-acids, to play here a very significant rôle. In this connexion the behaviour of aminohydroxy acids upon heating with hydrochloric and sulphuric acids was studied by us at different temperatures and for different periods of time. The experiment confirmed our anticipations, and we found that by changing these conditions the decomposition of aminohydroxy acids may be lessened a great deal.

The aminohydroxy acids put to the test were these:

1) Serine, obtained by cyanhydrine method. Analysis: nitrogen calculated, 13.33 per cent; found, 13.41. 2) Threonine, obtained by the method of West and Carter (1). Analysis: nitrogen calculated, 11.76; found 11.82 per cent. 3) β-Hydroxynorvaline, obtained according to West and Carter (5). Analysis: nitrogen calculated, 10.52; found, 10.52 per cent. 4) β-Hydroxinorleucine, obtained according to West and Carter (6). Analysis: nitrogen calculated, 9.52; found, 9.49 per cent. 5) β-Hydroxyvaline, obtained according to West and Carter (7). Analysis:

nitrogen calculated, 10.52; found, 10.48 per cent. 6) Isoserine, according to Fischer. Analysis: nitrogen calculated, 13.33; found, 13.40 per cent. 7) Hydroxyasparagic acid, obtained according to Dakin (4). Analysis: calculated hydrogen, 4.70 per cent; carbon, 32.2; nitrogen, 9.40; found hydrogen, 4.73 per cent; carbon 32.15; nitrogen, 9.38. 8) β-Hydroxyglutaminic acid, obtained according to

Table 1

Heating of Amino-β-hydroxy Acids with 25 per cent H_2SO_4

Name	Sample	t in °C	Time of heating, hrs	Ammonia N / Total N in %	Amino-hydroxy-acid N / Total N in %	Literary data Ammonia N / Total N in %
Serine	0.1965	125—130	16	3.97		8 hrs of heating with 25% H_2SO_4 14.3% t° not indicated
	0.1815	125—130	»	2.77		
	0.2529	140—145	»	4.50		
	0.1773	140—145	»	5.17		
	0.2040	160—165	»	6.90		
	0.2040	160—165	»	7.20		
Serine	0.0108	140—145	16	4.5 of serine	92.4 of serine	
Casein	0.1495					
Isoserine	0.2104	125—130	16	21.75		
	0.1557			21.38		
Threonine	0.1680	125—130	16	0.62		
	0.1680			0.43		
	0.2060	140—145	16	1.92		
	1.0000			1.62		
Threonine	0.5910	140—145	16	3.06		
Glucose	0.3980					
Threonine	0.5035	140—145	16	3.57		
Glucose	0.2432					
Threonine	0.5057	140—145	16	3.53		
Levulose	0.4725					
Threonine	0.0206	140—145	16	1.95 (threonine)	98.04	
Casein	0.1101			—		
β-hydroxynorvaline	0.4055	140—145	16	2.11		
	0.5873			2.27		
	0.4710		36	6.18		
	0.5060	160—165	16	3.9		
	0.5465			3.9		
β-hydroxyvaline	0.4658	125—130	16	5.43		
	0.2365	125—130	16	5.06		
	0.1722	140—145	16	12.56		
	0.5542			12.32		
β-hydroxyvaline	0.1066	140—145	16	15.02 (hydroxyvaline)	79.25	
Casein	0.0251					
β-hydroxy-norleucine	0.4415	140—145	16	2.87		8 hrs heating with conc. HCl 7.8%; t° not indicated
	0.1252	125—130	16	0		
Hydroxyasparagic acid	0.1860	—	—	0.68		
	0.9045	140—145	16	1.8		
	1.0000	—	—	1.49		
	0.5202	—	36	6.70		
	0.5522	—	—	6.38		
Do	0.0240	140—145	16	2.02 (hydroxyasparagic acid)	98.19	
Casein	0.1008					
β-hydroxyglutaminic acid	0.2020	125—130	16	14.0	78.8	16.5 hrs of heating with 25% H_2SO_4 12.5% (3)
	0.2132	125—130	16	14.3	78.4	
	0.2136	140—145	16	18.2	74.5	31.5hrs of heating 22% t° not indicated (3)
	0.2102	140—145	16	19.03	73.6	

Harrington and Randall ([8]). Analysis: nitrogen calculated, 7.01; found for the chlorhydrate, 6.94—6.96 per cent.

The experiments were run as follows. The samples of aminohydroxy acids listed in the tables were heated with 10 cm³ of a 25 per cent H_2SO_4 or with concentrated HCl for 16 or 36 hours at 125—130°, 140—145° and 155—160°C of the oil bath. After heating, ammonia, total nitrogen and aminohydroxy acid nitrogen were estimated in the solution by micromethods. The results of these tests are summarized in Tables 1 and 2.

Table 2

Heating of Amino-β-hydroxy Acids in 36 per cent Hydrochloric Acid

Name	Sample	t in °C	Time of heating, hrs	Ammonia N / Total N in %	Amino-hydroxy-acid N / Total N in %	Literary data: Ammonia N / Total N in %
Threonine	0.2095	125—130	16	2.97		8 hrs of heating in conc. HCl, t° not indicated, 7.8%
	0.2310	125—130	36	4.00		
	0.5000	140—145	16	4.21		
Hydroxyasparagic acid	0.3488	125—130	16	4.04		
	0.3284	125—130	16	3.96		
48% HBr	0.3767	—	2	15.10		
Hydroxyglutaminic acid	0.1683	125—130	16	25.5	70.8	

If the data on deamination of aminohydroxy acids; as found in these experiments, are compared with literary data, it is seen that certain temperature conditions may increase the stability of aminohydroxy acids upon hydrolysis.

The investigation of the homological series of amino-β-hydroxy acids made on serine, threonine, β-hydroxyvaline, β-hydroxynorvaline and β-hydroxynorleucine has shown that the extent of the decomposition of all of these aminohydroxy acids is directly controlled by the temperature and time of heating. The aminohydroxy acids of normal structure, with the exception of serine, were found to be stable upon heating with the 25 per cent H_2SO_4. The decomposition of serine reached 5 per cent at 140—145° C, i. e. was three times as low as that observed by Abderhalden and Broich. And it fell down to 3 per cent at 125—130° C. Heating with hydrochloric acid causes a somewhat stronger decomposition of aminohydroxy acids, but it can by no means be said to be high in this case either.

A much stronger decomposition was observed in the aminohydroxy acids having an isostructure. Thus, the decomposition of β-hydroxyvaline was as great as 5.43 per cent even at 125—130° C; and it attained 12.56 per cent at 140—145°C. At the same time the decomposition of β-hydroxynorvaline was only 2.27 per cent.

Somewhat unexpected results were obtained with β-amino-α-hydroxy dicarbonic acids. Contrary to expectation, it was found that hydroxyasparagic acid was not decomposed upon heating with mineral acids; while hydroxyglutaminic acid, in accordance with literary data, showed a very great per cent of decomposition. To prove in a final manner that β-amino- α-hydroxy acids are stable under ordinary conditions of protein hydrolysis, we have started experiments on their preparation from solutions after heating with acids.

I. 1.0035 g of threonine were heated with 20 ml of 25 per cent H_2SO_4 at 140—145° C for 16 hours. The solution was diluted with water and the sulphuric acid was removed quantitatively by a freshly prepared hot solution of Ba $(OH)_2$. The filtrate and the washings after condensation *in vacuo* were treated with absolute alcohol to nearly perfect dryness. The separated aminohydroxy acid was filtered off through a glass filter reduced to constant weight, and then dried in a vacuum

exsiccator. 0.9647 g of substance, or 96.16 per cent of the weight sample; were thus separated.

Analysis: nitrogen found, 11.69, 11.75 per cent; calculated, 11.76.

11.1 g of hydroxyasparagic acid was heated with 10 ml of conc. HCl at 125—130° for 16 hours. The hydrochloric acid was distilled off *in vacuo*; the residue was dissolved in water, acidulated with nitric acid and treated with a solution of $AgNO_3$ up to complete removal of chlorine ions. The filtrate and washings were treated with a concentrated water solution of 3 g of lead acetate. Afterwards the solution was made to show an alkaline reaction to lithmus and an acid reaction to phenol-phtalein, and the precipitate of the lead salt of hydroxyasparagic acid was separated and decomposed by hydrogen sulphide. The filtrate was condensed *in vacuo* to a thick syrup and treated with methyl alcohol. There was obtained 0.9372 g of the substance, i. e. 93.72 per cent of the hydroxyasparagic acid taken.

Analysis: nitrogen found, 9.28, 9.36 per cent; calculated, 9.4.

It is interesting to compare these data with the observations of Abderhalden and Broich. While decomposing serine with 25 per cent H_2SO_4 for 12 hours (temperature not indicated), they were able to separate only 65 per cent of the serine sample.

Thus; the stability of aminohydroxy acids on heating with mineral acids may be considered as well established for certain intervals of temperature. It is possible, however, that in the presence of proteins, and, especially, in the presence of carbohydrates, the decomposition of aminohydroxy acids is considerably increased. In view of this, special tests were made on heating aminohydroxy acids with carbohydrates and with casein.

The amino-acid sample was added with glucose, levulose or casein which were taken in quantities referred to in Table 1. The mixture was heated with a 25 per cent H_2SO_4 at 125—130° C or 140—145° C, for 16 hours. In the experiments with casein parallel tests were run on protein hydrolysis under the same conditions. In the two hydrolysates aminohydroxy acids were estimated by oxidation with sodium periodate. The difference between the estimations made in the mixture of aminohydroxy acids with casein and those made in a pure casein represented the quantity of the unchanged amino-acid. The quantity of ammonia was likewise determined from the difference.

It is evident that the addition of carbohydrates has little influence on the stability of aminohydroxy acids. Thus, on heating threonine with glucose or levulose for 16 hours the quantity of ammonia nitrogen increased from 1.92 to 3.5 per cent. Nor has the addition of casein any influence on threonine or hydroxyasparagic acid. Deamination of serine and β-hydroxyvaline shows a very slight increase, too. However, from the amount of the aminohydroxy acid found; the decomposition of serine attains 7.6 and that of hydroxyvaline 20.75 per cent. It is very likely that these amino-acids take part in the formation of humine substances.

It is interesting to draw a comparison between the degree of decomposition of aminohydroxy acids and of some other amino-acids which are usually determined in protein hydrolysate (Table 3). It will be found that in a number of cases the aminohydroxy acid is the more stable.

The disagreement of the results obtained by different authors in aminohydroxy acid extimations and the fact that these acids have been scantily investigated are due to the circumstance that the conditions of the hydrolysis were not fixed accurately and no perfect method for the investigation of aminohydroxy acids has yet been devised.

On the basis of the data obtained we have made estimations of β-amino-α-hydroxy acids in casein and gelatine. When casein had been heated with 25 per cent H_2SO_4 at 125—130° C for 16 hours the amino-β-hydroxy acid nitrogen made up 8.50 per cent of the total nitrogen. This value dropped to 7.5 per cent when the temperature was increased to 140—145° C.

The results obtained coincided with the data by Macara and Plimmer [10] who after hydrolysing casein with 20 per cent HCl during 24 hours found 8.05 per cent

Table 3

Decomposition of Different Amino-acids upon Heating with Mineral Acids

Name	Conditions of experiment			NH_2/N in %	Estimation of amino-acids in %
	t°	time in hrs	acids		
Threonine	125—130	16	25% H_2SO_4	0.62	—
» + glucose	140—145	16	»	3.06	—
» »	125—130	16	conc. HCl	2.97	—
» »	not indic.	24	20% HCl	3 [12]	—
» +casein	140—145	16	25% H_2SO_4	1.95	98
Phenyl-alanine	125	2	8% H_2SO_4	16 [9]	—
»	125	2	20% HCl	8 [9]	—
Edestine + 10% sucrose, found:					
arginine	—	—	—	—	84
histidine	—	—	—	—	87
tyrosine	—	—	—	—	36
β-hydroxyvaline + casein	140—145	16	25% H_2SO_4	15.02	79.25

of amino-β-hydroxy acid nitrogen in the casein. The content of β-amino-α-hydroxy acid nitrogen in gelatine was 3.9 per cent of the total nitrogen.

Conclusions

1. β-amino-α-hydroxy acids of the fatty series show but a slight decomposition when heated with mineral acids within certain temperature intervals. Amino-hydroxy acids of the isostructure decompose stronger than nor-aminohydroxy acids.

2. The addition of carbohydrates or of casein has an insignificant influence on the decomposition of aminohydroxy acids. An exception is formed by serine and β-hydroxyvaline which show as high a decomposition as 7.6 and 20.5 per cent. respectively.

3. Amino-β-hydroxy acids have been estimated in casein and gelatine.

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Received
22. VI. 1943.

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1943. Volume XLI, № 2

GEOLOGY

ON THE CLASSIFICATION OF OIL RESERVOIRS

By P. P. AVDUSIN and M. A. ZVETKOVA

(Communicated by S. S. Nametkin, Member of the Academy, 10 V 1943)

Among the factors determining the nature and scope of the hydrodynamic processes going on within the reservoir rocks of petroleum deposits, the petrographic properties of the reservoirs and their mineralogical composition and structure play an outstanding part.

The experimental work carried out during 1936—1937 at the Institute of Mineral Fuel of the Academy of Sciences of the USSR by P. P. Avdusin, V. P. Baturin and Z. V. Varova showed that sands consisting of quartz grains have the best filtrating capacity: their fragments are the most isometric, and the surface has the least adsorption capacity. The existence of cleavage fissures in feldspars and diopside and the indented habit of the grains of these minerals reduces the filtrating capacity of the sands consisting of them. Still lower filtration coefficients were shown by sands consisting of rock fragments: porphyrites, clay shales, phyllites, etc.

The filtration rate is also greatly affected by admixtures of mica and minerals of the clay group.

Aside from the phenomena of film tension affecting the rate of migration of oil in the rock, at the contacts between the solid polymineral phase of the reservoirs and the liquid oil, the processes of catalysis and adsorption sometimes bring about the segregation from the oil of solid components such as resins, asphalts, etc.

The formation of the latter in the pore channels reduces the effective cross-section of the pores and thereby contributes to the decrease of the velocity of the oil flow.

The structure of the pore spaces of the reservoir plays an exceedingly important part in determining the nature of the movement of oil through a porous rock. Getting into the porous body of the reservoir, the oil fills up all the communicating voids, small channels and cracks of the rock. The total volume of these spaces determines the complete porosity of the rock. The mass of oil per unit volume of the reservoir determines the coefficient of oil saturation. When circulating over the bed, not the whole mass of oil migrates, but only the portion filling more or less wide pore channels of the reservoir, while the oil filling capillary and subcapillary fissures and voids does not take part in the movement of the oil flow.

In studying the structure of the reservoirs from the standpoint of an economic evaluation of the oil deposits, it is important to know the volume and morphology of those channels along which the main masses of fluid is moving.

We have developed a method for studying the structure of these effective (for a given fluid) channels. The principles of this procedure consist in the follow-

ing: a specimen of the reservoir rock to be studied, all possible care being taken to preserve the original structure of the rock, is saturated under vacuum with bakelite (bakelite resin); the latter is heated to a temperature at which the viscosity of bakelite corresponds to that of the oil from the deposit investigated. The specimen of the rock soaked with bakelite is dried; from this specimen in a direction normal to the fluid flow, a thin plane-parallel section is prepared which is further on examined under the microscope.

With the aid of an integration table of Chand or a push-integrator designed at the Institute of Economic Mineralogy, the cross-sectional area of the channels filled with coloured bakelite is calculated; it is the ratio of this area to the total area of the preparation to be investigated that determines the effective porosity of the rock P_e.

As a feature determining the nature of the movement of oil in the pore channels of the reservoir rock, P. Avdusin proposed to consider the quantity Φ which is inverse to the ratio of the section of pore spaces normal to the fluid flow to the perimeter of the equivalent ideal cylindrical channel*. The method for the determination of this value has been described in a previous paper (1).

The coefficient Φ determining the morphological peculiarities of the pore spaces of the reservoirs, combined with P_e determining the volume of the spaces filled with the oil migrating in the bed, enables us to approach the solution of the problem as to the productivity of the bed and the economic evaluation of the field: P_e determines the fluid mass taking part in the movement along the bed, and Φ characterizes the rate (dynamics) of the migration of this mass.

Collections of specimens of rocks—oil reservoirs, in which the structure of the pore space was studied, were analysed also for their permeability. The results of such investigations of reservoir rocks point to a definite relation between the permeability of the rocks and their structural parameters: the larger P_e and Φ, the higher the production of the bed; with a decrease in P_e the mass of the filtrating oil also decreases; and with a decrease of Φ increases the resistance of the filtrating flow.

The experiments now in progress at the Institute of Mineral Fuel show that, according to the structure of the pore spaces of the reservoir rocks, and the value of the active factor (pressure drop), the nature of the movement of the fluid in the porous body of the reservoir may vary, too.

During 1938—1941 we investigated typical representatives of reservoir rocks of oil fields in Azerbaidjan, Georgia, North Caucasus, Ural-Emba and Ural-Volga regions, Ukhta region, etc. A comparison between the petrographical properties of reservoir rocks of the oil field studied and their permeability; as well as data furnished by the exploitation of oil-bearing horizons, makes it possible to class all reservoir rocks into several groups.

This classification of reservoirs is based upon structural features—volume (dimensions) and shape of their pore spaces. Using these features, all reservoirs may be divided into five classes (denoted by the first letters of the Latin alphabet, A, B, C, D, E), characterized by the following values of effective porosity P_e.

1. Class A—reservoirs with	$P_e > 20\%$	Reservoirs of large capacity
2. Class B »	» $P_e = 15 \div 20\%$	
3. Class C »	» $P_e = 10 \div 15\%$	Reservoirs of medium capacity
4. Class D »	» $P_e = 5 \div 10\%$	
5. Class E »	» $P_e < 5\%$	Reservoirs of small capacity

Each of these five classes may be in turn subdivided into three groups according to the shape of the pore spaces of the reservoirs:

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* $\Phi = \frac{0.2 \cdot \pi^{3/2} R^2 \sqrt{P_e}}{\Sigma l K} \cdot 100$, where R is the radius of the drawing in cm; K, magnification of the drawing; Σl, total perimeter of the effective pores of channels in the plane of the drawing in cm.

1st group, reservoirs having a more or less isometric section of the pore channels, and hence making possible considerable filtration velocities. $\Phi > 0.20$*.

2nd group, reservoirs with $\Phi = 0.10 \div 0.20$.

3rd group, reservoirs characterized by a complex structure of their pore spaces and a large resistance to the moving oil flow. $\Phi < 0.10$.

The study of individual strata of the reservoirs of some oil fields of the Soviet Union shows that the structure of the reservoir of oil deposits is sometimes a very complex one and that the oil deposits are not infrequently confined to a number of intercalations differing in their physical properties, and each of them requiring an individual approach when selecting rational exploitation methods.

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* With $K = 50$ and $R = 7.5$ cm.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

PLANT PHYSIOLOGY

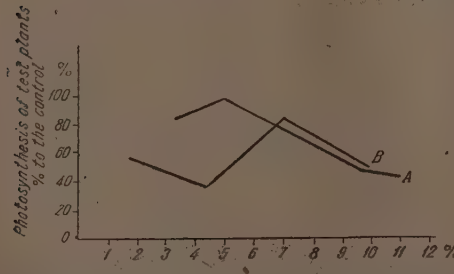
RATE OF PHOTOSYNTHESIS AS AFFECTED BY THE METHOD OF DEHYDRATING THE ASSIMILATIVE TISSUE*

By V. A. BRILLIANT

(Communicated by L. A. Orbeli, Member of the Academy, 19. VI. 1943)

In studying the dependence of the photosynthesis upon the amount of water in the assimilative tissue, the action of several substances causing withdrawal of water, viz., sugars or salts, has often been used alongside of direct drying. From literary data it may be concluded that the influence of osmotically active solutions on the photosynthesis is in general similar to the effect of direct drying; in both cases a certain degree of dehydration, mostly even a slight one, increases the rate of photosynthesis in comparison with the control plant saturated with water.

But it must be taken into account that the physiological effect of drying may be different from that caused by the osmotic withdrawal of water, and that each may have some specific features. Plasmolysis, for instance, is observed when hypertonic solutions are used; when the tissue is dried in the usual way, air can penetrate inside the cell, and so on. In this connexion it is interesting to investigate the influence of the various methods of dehydration on the photosynthetic activity of plants.



Photosynthesis of *Elodea densa* in relation to water withdrawal by drying (*A*) and by salt treatment (*B*)

The experiments reported here included two different sets. In one of them shoots of *Elodea densa* were dried in the air or in an exsiccator over a solution of sulphuric acid of a definite concentration. In the other set the shoots were kept for some time in a solution of sodium chloride 0.1—0.2 g/mol. In both sets the loss of water was determined from the decrease of the shoots in weight. The control plants were kept in water. When the dehydration period came to an end, the plants in both sets were transferred to water, and the influence of the dehydration was thus studied from its after-effect upon the assimilating cells. The exposure was made in tetrahedral vials with ground-in stoppers. In order to keep temperature at a constant level and to secure a uniform illumination, the vials were placed along the radii of a circle in a large bath with running water. The plants received light from a 1000 watt lamp suspended over the vials at a distance of 50 cm. The rate of photosynthesis was determined

* A. J. Kologrivova and N. N. Chizhova have taken part in this investigation.

according to Winkler from the content of oxygen in the water. All tests were run in at least two replicates.

The results of the tests are brought out by the curves of the figure. They show that with small loss of water (e. g. 5—6 per cent of the raw weight of plant) the salt (*B*) causes a stronger reaction of the rate of photosynthesis, than does the drying (*A*); the difference is smoothed out when the loss of water is high.

A very similar result was obtained in other experiments made with *Potamogeton praelongus*. Some leaves of this plant were dried in the air, while the neighbouring leaves were subjected to the action of a cane-sugar solution 0.3 g/mol.

In Table 1 the results of two of these tests are given. In the first of them an equal loss of water gave a lower reduction of the rate of photosynthesis in the case of drying than in the case of osmotic withdrawal of water. In the second test the rate of photosynthesis diminished to an equal extent, whether the plants had lost 11 per cent of their water as a result of air drying, or 7.7 per cent by cane-sugar treatment.

Table 1

Leaf	Method of dehydration	Loss of water in % to raw weight	Photosynthesis	
			mg O_2 per hour per 1 g of dry weight	% to control
Control .	—	—	12.90	100
Test a .	Air drying	7.8	6.02	47
» b	Sugar solution	7.8	3.75	29
Control .	—	—	9.96	100
Test a .	Air drying	11.0	4.81	48
Test b .	Sugar solution	7.7	4.05	41

It appears that the dehydration of the assimilative tissue by means of salt has a different physiological action from that caused by the loss of water through evaporation. As the plants were kept in the salt solution for a short interval of time (from 20 to 60 minutes) and the results were invariable when the sugar solution was used, there is good reason to ascribe the change in the rate of photosynthesis to the loss of water rather than to a direct action of the salt introduced into the cell. To explain the fact that the rate of photosynthesis is differently affected by osmotically active solutions and by air drying, one may suppose that the different mechanism of water withdrawal brings about different alterations of the cell colloids which, in the first place, affect the photosynthetic reactions of the plant to darkness.

Very interesting from this point of view are the results obtained by subjecting *Elodea densa* to the action of calcium salts which penetrate into the cells with difficulty. In part of these experiments the *Elodea* shoots were kept in a solution and afterwards were transferred to water for exposure. In the other experiments the exposure took place in the salt solution, and in order to avoid a possible error in the procedure the same solution was used in the blank test. As shown by the figures of Table 2, a sharp rise of the rate of photosynthesis as compared to control was observed in the second set. Such an effect was not observed in the case of sodium salts, however.

The explanation that suggests itself is that the calcium ion has some specific action upon the colloid of the protoplasm. A reduction of hydration and increase of viscosity of the colloids under the action of calcium was reported by several authors ([5], [6]).

I was able to show in an earlier work ([1]) that when the rate of photosynthesis increases in response to a certain degree of dehydration of the assimilating tissue, this is due in the first instance to the darkness reactions of the process. This fact

Table 2

Treatment			Rate of photosynthesis					
			Exposure in water			Exposure in solution		
Salt solution in g/mol		Time in min.	mg O_2 per hour per 1 g of dry weight		% to control	mg O_2 per hour per 1 g of dry weight		% to control
			test	control		test	control	
$Ca(NO_3)_2$	0.15	20	3.38	3.06	110	48.68	2.92	640
	0.20	2	1.31	1.60	82	—	—	—
	0.20	15	—	—	—	20.76	5.10	407
	0.20	20	—	—	—	9.97	4.27	233
	0.20	20	2.00	4.00	50	12.61	1.60	788
$CaCl_2$	0.15	20	1.43	3.06	47	25.85	2.92	885
	0.20	20	−6.62	4.00	0	—	—	—
	0.20	20	−7.53	3.22	0	15.56	3.22	483
$NaNO_3$	0.20	20	—	—	—	0	1.60	0
	0.40	20	−6.78	4.27	0	—	—	—
NaCl	0.15	20	3.22	3.80	85	2.90	2.89	100
	0.25	20	3.28	3.80	89	0.77	2.89	27

I then brought to comparison with the data by Sissakian and Kobiakova ([3], [4]), according to whom a dehydration of leaves not exceeding a certain limit causes a considerable increase in the synthetic activity of several different enzymes. In spite of the differences in the character of the enzymic reactions involved in the photosynthesis, on the one hand, and of the reactions determined by the work of invertase, protease etc.; on the other, it may be supposed that the effect of the dehydration of assimilative cells on photosynthesis is connected with its influence upon the trend of enzymic processes, which then make for synthesis. In this light the sharp rise of photosynthesis under the action of the calcium ion which is known to be an energetic activator of certain enzymes and, particularly, a catalyst of certain synthetic reactions ([2]), finds its explanation.

Conclusions. 1) The dehydration of assimilative cells by osmotically active solutions has, in a number of cases, a stronger influence on photosynthesis than direct drying, even if the loss of water is the same. 2) Calcium salts in hypotonic concentration cause a sharp increase of photosynthesis in *Elodea densa*.

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Received
1. VI. 1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

PHYSIOLOGY

METHYL-NAPHTHAQUINONE (VITAMIN K_3) AS USED FOR ARRESTING HAEMORRHAGES AND FOR WOUND TREATMENT

By A. V. PALLADIN, Member of the Academy

Until recently the application of vitamin K to medical practice has been a strictly limited one. It was assumed to be safely established ([1], [2]) that both the natural vitamin formed in plants (K_1) and that formed by bacteria (K_2), as well as their simpler, though more active synthetic analogue, 2-methyl-1; 4-naphthaquinone [for which I suggested the name of vitamin K_3([3])], can arrest only haemorrhages due to a reduced content of prothrombin in the blood (hypoprothrombinaemia) and to a low rate of blood coagulation connected herewith. Vitamin K and its analogues have therefore up to the present time been applied largely to the control of haemophilia and of the underlying hypoprothrombinaemia in cholaemic patients, and to the treatment of such haemorrhagic phenomena which, being independent of icterus, are attended with hypoprothrombinaemia. To these belong haemorrhages occurring in certain cases of sepsis; in the case of stomach ulcer, typhoid fever, haemorrhagic diathesis of sucklings, and some gynaecological haemorrhages. In accordance with the above-said ([4]) it was assumed that preparations of vitamin K are not to be used, if we have to do with a normal prothrombin content in the blood; their haemostatic effect being nil in this case.

The aim of the work started by me after the beginning of the Great National War in co-operation with B. I. Goldstein, R. I. Lirzman, B. I. Khaikina and E. J. Milgram of the Biochemical Institute of the Academy of Sciences of the Ukrainian SSR, was to test whether 2-methyl-1; 4-naphthaquinone; as the most active among the vitamins K; will also stop such haemorrhages which are not connected with a reduced prothrombin content in the blood. In other words, it was to be tested for its efficiency against various parenchymatous traumatic haemorrhages due either to wounds or to surgical intervention; or else to affections attended with vessel injuries (as for instance; pulmonary tuberculosis).

Our investigations were based on material supplied by cliniques and evacuation hospitals of the town Ufa; under co-operation and clinical control of a number of clinicists, surgeons and therapeutists*. 2-methyl-1, 4-naphthaquinone was chiefly applied in the form of the preparation manufactured under my supervision and control at the Ufa vitamin factory, and sold by the latter under the name of «vitamin K_3».

It was always administered *per os* in the form of alcohol solutions; this mode of application which guarantees for a good absorption of the drug ([2]) was found by us to excell in many respects both intramuscular injections of oil solutions, and

* It is our pleasant duty here to express our thanks to Professors B. E. Linberg, A. A. Polianzev, I. G. Lukomsky, A. S. Morozovsky, S. V. Miknailovsky, A. M. Agaronov and V. J. Karnaukhov, who helped us in our work.

administration of the latter *per os*. The patients were given 10—15 mg of methyl-naphthaquinone in alcohol solution *.

The possibility of arresting with the aid of vitamin K_3 such haemorrhages as are not dependent on a reduced prothrombin content in blood was evidenced in the first line by its successful application in the case of bleeding from granulations and of haemorrhages due to perforating wounds of the thorax. In a number of cases haemorrhages which persisted during several months after the trauma, defying all usual treatments; stopped after alcohol solution of methyl-naphthaquinone was taken, usually after 2—3 applications of the preparation at a dose of 10—15 mg. Prothrombin content and rate of blood coagulation in these patients were normal, as a rule.

Vitamin K_3 was found to be applicable; moreover, for stopping and preventing haemorrhages accompanying various surgical procedures. Thus, when applied prophylactically previous to tonsilectomy, methyl-naphthaquinone not only diminished bleeding in the course of the operation, but reduced post-operational bleeding, as well. This made it possible surgically to intervene in such cases of tonsilitis which had been considered inoperable before the introduction of vitamin K_3. About a half of the patients suffering from tonsilitis showed a normal prothrombin content of blood, while in another half it was low; in these it came up to the normal level after they had taken vitamin K_3.

Vitamin K_3 gave also an excellent effect when applied in the course of surgical treatment of maxillary sinusites due to fire-arm wounds, and accompanied with osteomyelitis of the upper jaw. Surgical procedures (sequestroantrotomy) are usually followed in this case by profuse post-operational bleeding. By giving vitamin K_3 prophylactically at a dose of 30 mg before the operation, the haemorrhage during and after the operation could be reduced considerably. It was found possible to avoid the tamponage of the cavity as a method for arresting the haemorrhage.

Haemorrhoidal bleeding, however prolonged and exhaustive, has nearly always been stopped by the application of vitamin K_3. It produced also a good effect in the case of epistaxis; and in both cases the patients showed a normal content of prothrombin in their blood.

The positive effect produced by methyl-naphthaquinone in the case of tuberculous haemophthisis is a quite conspicuous one, which permits to apply it instead of calcium chloride.

Vitamin K_3 also stops gynaecological haemorrhages; and not only such as are connected with a reduced prothrombin content in the blood, but also bleedings which are not caused by hypothrombinaemia, as e. g. uterine haemorrhages and those accompanying endometritis with inflammation of the annexes, essential bleedings, etc.

The first important result of our investigation was thus to have established the possibility to stop and prevent with the help of vitamin K_3 various parenchymatous and capillary haemorrhages in patients showing a normal content of prothrombin in their blood.

The second important result was the discovery of the accelerating effect of vitamin K_3 upon the healing of wounds. On an abundant clinical material has been proved the possibility of accelerating with the aid of vitamin K_3 the healing of persistent superficial sores with luxurious, bleeding granulations, as well as of indolently granulating persistent wounds, defying all usual treatments, and of fresh extensive, more or less superficial, wounds. No less successful was the use of methyl-naphthaquinone for the treatment of bed-sores, frost-bites and burns, as well as of ulcers, e. g. of ulcerous stomatitis.

The data obtained make us renounce to the interpretation given by Dam [1] to the mechanism of the action of vitamin K. This writer believes that the rôle of vitamin K is to affect the formation of prothrombin; consequently, vitamin K

* A 0.1 per cent alcohol solution of methyl-naphthaquinone was prepared, which previous to internal use was diluted with an equal volume of water.

and its analogues are supposed to be able to arrest bleeding in the case of hypoprothrombinaemia only. Our investigations have shown that vitamin K_3 arrests haemorrhages in patients with a normal prothrombin content in their blood as well. The effect of vitamin K_3 is, therefore, not restricted to the formation of prothrombin. In a number of cases we observed a reduced rate of blood coagulation along with a normal content of prothrombin. The application of vitamin K_3 restored the normal rate of blood coagulation, which points to the fact that, besides prothrombin, vitamin K_3 affects some other substances participating in the process of blood coagulation, possibly the fibrinogene.

The faculty of methyl-naphthaquinone to accelerate the healing of wounds, i. e. to stimulate regeneration processes, and its efficiency in the case of scurry haemorrhages, point to the influence of vitamin K_3 upon the endothelium. Along with this, pathological-histological investigations carried out at our suggestion by A. I. Smirnova-Zamkova and B. I. Khaikina have shown that vitamin K_3 influences the elements of the active mesenchyme of the connective tissue.

Thus, data are being accumulated that may serve to build a new theory of the mechanism of action of vitamin K instead of Dam's theory which ought to be abandoned, as being in contradiction to the material available. At the time being, it may even be harmful, for it imposes limitation upon the use of the analogues of vitamin K; and of vitamin K_3 (methyl-naphthaquinone) in the first line.

From what has been said above, it is evident that vitamin K_3 (methyl-naphthaquinone) deserves a wide-scale application in hospitals and other medical institutions, and not only in the rear, but at the front as well. It must be used as a means against parenchymatous and capillary haemorrhages consequent to wounds; and for the treatment of wounds, in order to restore rapidly the health and fighting capacity of the valiant defensors of our motherland.

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Received
11. VI. 1943.

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EXPERIMENTAL MORPHOLOGY

REGENERATION OF THE ECTODERM OF THE GILL-ANLAGE IN AXOLOTL AT THE STAGES OF THE EARLY, LATE AND ELONGATED TAIL-BUD

By F. MURTASI and M. SOSNINA

(Communicated by L. A. Orbeli, Member of the Academy, 21. VI. 1943).

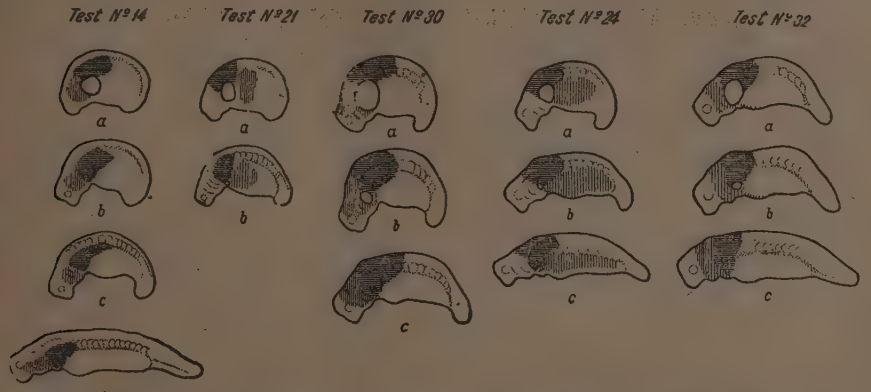
In his research on the development of the external gills in *Amblystoma punctatum* Severinghous (1) has observed in one of his experiments a case of the absence of regeneration of external gills at the stage of the late tail-bud. This sole experiment served afterwards as a starting point for an investigation made by F. Murtasi on the regeneration of the external gills and their anlage in amphibiae during their development. By means of special experiments (3) on axolotls and certain frogs (*Rana ridibunda* and *Rana temporaria*) this author was able to establish the presence of a critical period in the development of the external gills in these amphibiae. He found that the regenerative effect of the external gills in axolotl, *R. ridibunda* and *R. temporaria* is strongly reduced at the critical period; after which it gradually increases again (but not fully) up to the stage of the outwardly perceivable anlagen (cones) of the external gills. After this stage the regenerative effect decreases again. It was also found out that the critical period of the development of the external gills corresponds in *R. temporaria* to the stage of the tail-bud becoming visible; in *R. ridibunda*, to the stage of the early tail-bud; and in axolotl, to that of the late tail-bud.

Further on, Mashkovtzev (2), while studying the determination of the external gills in amphibiae, has also established the absence of regeneration of the external gills in *Rana esculenta* (after removal of the gill ectoderm) at the stage of the late tail-bud. As a matter for regret, Mashkovtzev did not continue his experiments beyond this stage; he merely suggested the idea that the absence of regeneration of the external gills in *R. esculenta* at the stage of the late tail-bud is due to the fact that, being removed, the ectoderm of the gills is replaced by that of the head spreading over the wound surface of the anlage of the gills. At that time the latter is already determined as epithelium of the tegument, and gives therefore no response to the inductive action of the ento-mesoderm of the gills (cf. to the results of the experiments by Harrison who substituted the ectoderm of the trunk for that of the gills). This supposition (not proved experimentally) calls forth, in the first line, the question about the source of the material used for the renewal of the removed ectoderm of the gills. But at the same time, the supposition made by Mashkovtzev only for the case of *R. esculenta*, may be extended in principle also to the other representatives of amphibiae, accounting for the failure of regeneration of the external gills at some definite periods of development. In this case the absence or a strong decrease in the regenerative effect of the external gills in amphibiae at the time of the critical period may be interpreted as a result of inhibition of the regenerative potency of the gill-anlage by the differently determined ectoderm. Such a substitution of the removed ectoderm must evidently be peculiar

only of the critical period, since before and after it the regeneration of the external gills takes place in amphibiae under experiment (axolotl, *R. ridibunda*, *R. temporaria*).

In order to elucidate all these questions we undertook a series of experiments on axolotls. The experiments included 3 stages of development: the stage of the early tail-bud, that of the late one; and the stage of the elongated tail-bud (stages 25, 29 and 33 according to Harrison).

The method of this experiment was a very simple one: the ectoderm directly adjoining the anlage of the gills was stained neutral-red and nil-blausulphate by means of small pieces of agar soaked in these dyes. In one part of the experiments we stained the ectoderm adjoining the gill-anlage in front and from above, and in the other part that adjoining it from behind and from above. After these parts of the ectoderm had been duly stained, the pieces of agar were taken away, and the non-stained part of the gill ectoderm was removed without injuring the mesoderm.



Graphic records of the experiments illustrating the process of regeneration of the removed gill ectoderm: *a*, the initial stage, at which the ectoderm adjacent to the gill-anlage was marked, and the non-stained gill ectoderm removed; the outlined field is the surface of the wound produced by the removal of the gill ectoderm: *b*, *c*, *d* are the subsequent stages of development. The field hatched with vertical lines is the neutral-red mark, and the one with horizontal lines is the nil-blausulphate stain

The result of this operation was an even wound surface at the place of the ectoderm of the gill-anlage, delimited in front and above, or behind and above by the adjoining stained ectoderm (fig. *a*, experiments Nos. 14; 21; 30, 24 and 32). Subsequently such experimental embryos were systematically subjected to a thorough graphical recording by means of an ocular micrometer set up in a binocular dissecting microscope. The total number of experiments amounted to 32. The results of all the experiments were so much of the same type that we give here but a few illustrations of our experiments (see figure).

When marking the ectoderm adjoining directly the gill-anlage in front and above at the stage of the early tail-bud, we may see that the removed gill ectoderm is replaced by that situated above the gill-anlage, the latter gradually sliding downwards (in ventral direction) upon the wound surface of the gill-anlage (exp. No. 14).

The staining of the ectoderm adjoining the gill-anlage from behind and from above shows that the healing of the wound surface of the gill-anlage proceeds also mainly at the expense of the ectoderm above the gill-anlage. The ectoderm adjoining the gill-anlage from behind takes but a small part in the regeneration of the gill ectoderm (exp. No. 21).

In reality an identical process of substituting the removed ectoderm of the gill takes place also at the stages of the late tail-bud (exp. Nos. 30, 24, 32). At these stages; as at the preceding ones, the removed ectoderm of the gill is

regenerated mainly at the expense of the ectoderm above the gills. It is true that at these stages the part taken in the regeneration of the gill ectoderm by the ectoderm situated directly behind the gill-anlage is manifested more clearly. The regeneration of the ventral part of the ectoderm of the gill-anlage proceeds at the expense of this ectoderm, as it is shown by the graphical record of exp. No. 24. Unlike this, the head ectoderm, that is to say the ectoderm adjoining the gill-anlage directly on the front, takes in this case too no part in the regeneration of the gill ectoderm.

Thus, at all the investigated stages of development in axolotl the regeneration of the ectoderm of the gills proceeds mainly on account of the ectoderm situated directly above the gill-anlage, and only at a small degree, at the expense of the ectoderm adjoining it from behind. As to the ectoderm of the head, adjoining the gill-anlage in front, it takes no part in the regeneration of the ectoderm of the gills. The stage of the late tail-bud being the critical period for the gill-anlage, we may therefore conclude that the way of regeneration of the ectoderm of the gills before and after the critical period, as well as during the critical period of the gill-anlage itself, is identical, of the same type. And as the regeneration of the external gills takes place both before as after this critical period, it is quite evident that the absence or the decrease of the regenerative effect of the external gills in amphibiae at the critical period is due not to the influence of the differently determined material substituting the removed material of the gills, but to the peculiar nature of the critical period itself. And as the general character of the course of regeneration of the external gills is similar in axolotl, *R. ridibunda* and *R. temporaria*, there is some probability that our data concerning the axolotl will prove true also for the *Anura*.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

HISTOLOGY

GROWTH AND TRANSFORMATIONS OF GLIO-EPENDYMAL ELEMENTS OF THE SPINAL CORD OUTSIDE THE ORGANISM

By N. G. CHLOPIN

(Communicated by L. A. Orbeli, Member of the Academy, 19. VI. 1943)

Proceeding in our study of the various tissues of ependymal-glial type by the method of explantation, our attention has been naturally attached to the most ancient region of the central nervous system, viz. to the spinal cord. Our previous data ([2-4]) have shown that, in spite of many features they possess in common, neuroglia of different regions of the brain (optic ways and hemispheres) display a rather stable specialization in different directions. The question naturally arises, to what degree the results obtained with the white matter of the hemispheres may be extended to the spinal cord, and to its grey matter, in particular. It seemed besides highly desirable to add data secured with a new material to those already accumulated on the ependyma from the elements of which I still failed to obtain extensive growths.

In search for material our choice fell upon the region of the grey matter of the spinal cord, white matter being subject to early myelinization of many of its bundles, and with the method now available the presence of myelin is a great inconvenience in cultivation. Besides, by removing the peripheral parts of the spinal cord it was possible to get rid of the meninges, the growth of whose elements might, according to the data obtained at this laboratory, obscure the interpretation of the pictures observed. Moreover, by explanting grey matter in the first line, we were likely to find the ependyma in a sufficient part of the cultures. Material for sowing was taken from newborn rabbits up to 5—6 days old. Like the rest of regions of the nervous system, the best results were obtained with the spinal cord when the medium was added very small quantities of the extract. Sowing was practised by the method of a suspended drop. The cultures were being studied for 30—40 days outside the organism; passages were made regularly, mostly on the seventh day, instead of the third or fourth day, as usual.

Under the conditions of my experiments which were focused upon the investigation of the subsidiary components of the spinal cord, there was, as a rule, no growth of nerve fibres or any other progressive phenomena in the neurons. Neither could I observe anything that would resemble active migration of the neurons into the zone of growth. The neurons soon became subject to distrophic and degenerative alterations upon which I will not dwell here. In most of the cases they perished after 1—2 weeks leaving no traces at all. After this period the central bits which during the early stages of cultivation had become appreciably smaller and rounder, invariably assumed the typical structure of solid glial massif as already described by me for the optic ways and for the wall of brain ventricles. The cytoplasm of the cells of these massif usually shows differentiation of a dense network of thick glial fibres which stain deep black with iron haematoxylin.

The distinctions in size, shape and dyeability of the glial nuclei faded away, and the massifs became homogenous. These observations illustrate with sufficient clearness the differentiation of protoplasmic astroglia, and apparently also of oligodendrogliae and even orthegagliae into fibrous astrogliae, which does not take place in the grey matter of the spinal cord under normal conditions. There is, therefore; every reason to interpret the orthegaglia as a variety of neurogliae rather than cells related to the histiocytes of the connective tissue.

As in the case of the white matter of the cerebral hemisphere, at different stages of cultivation two main types of extensive growth could be observed, usually accompanied by the migration of amoeboid macrophage-like glial cells and by an intense liquefaction of the fibrin-clot. As a result of extensive growth, a whole complex was formed of structures typical of, though poorly differentiated for the neuroglia of the brain regions referred to above: viz., of thin cellular membranes and relatively thick strata, strands, as well as loosened reticulate cellular bodies and isolated cells with processes, sometimes of «fibroblast-like» shape. The elements of these growths usually showed appreciable or even high mitotic activity. Along with glial growths quite different pictures could be observed in the explant, and not only at early stages, as in the culture from the wall of lateral ventricles, but not infrequently at more advanced stages (after 20 days and more); as well. Within the zone of growth, mostly at the surface of the mica, in the region of liquefaction cavities, cells appeared with thin, long, thread-like, not infrequently branching processi. As a result, the zone of growth assumed the character of a complicated and peculiar network, made up of the cellular bodies and their interconnected processi. Part of these elements were capable of perfect isolation; showing in all details a complex «multipolar» or «bipolar» structure of their cellular body. These cells were of the nature of astrocytes, or looked even more or less like neurons. They could therefore be denoted as neuron-like elements. Part of them were remarkable for their relatively big size, and grew bi- or mononuclear as a result of constrictions. The mitotic activity of these cultures was not high, as a rule. In some cases no mitoses at all could be observed. The appearance of neuronoid structures within the zone of growth and at such stages of cultivation when the whole central bit had already been transformed into a homogenous glial massif, while the neurons which had originally made part of the explant, vanished without having left any traces, clearly points to the fact that neuronoid cells are astrocytic elements. It was no difficult matter, besides, to ascertain by way of direct observation that the neuronoid structures are formed at the expense of the neuroglial elements of the central bit. They should be interpreted as a result of a certain histotypical differentiation of glial elements within the zone of growth of the tissue cultures, and as the manifestation of some histoblastic potencies specific for the neurogliae of a number of sections of the central nervous system. It is well known that many astrocytic elements of a normal spinal cord or brain may well compare as to their appearance, complex shape of cellular body and length of processi with some nerve cells. This is seen with particular clearness on macerated preparation studied by many older writers, and among others widely used by Gierke (1).

The data secured in the course of the present work and of a preceding work of mine, as well as the study of literary data, show that these cells supplied with thin processi which were described by several writers as neurons migrated within the zone of growth, are, as a matter of fact, astrocytic neuroglial elements, in so far as cultures from a more or less differentiated nervous system are concerned. The possibility of process-bearing neuroblasts differentiating from extensively growing non-differentiated epithelium-like elements, and from the so-called medulloblasts of the embryonic nervous tube cannot be excluded. However, the data available on this point are open to dispute, since in this case the differentiating astrocytic elements may possibly be mistaken for neuroblasts. When grown outside the organism, the particles of the nervous system at various stages of the latter's development, offer a rather complex and intricate picture. This picture defies correct interpretation unless the histoblastic potencies of the tissues of ependymal-

glial type are thoroughly and critically investigated by the explantation method. It is the lack of such an investigation that is responsible for the fallacious conclusions made by many an author working in this domain.

Extensive growth of the ependyma could for the first time be observed in explants from the spinal cord, which contained lining of the myelocoel. The ependymal cells which originally show a lively oscillatory movement of their ciliae were seen to advance from the central bit on to the zone of growth, forming complex strata and strands; these could by and by be distinguished from the surrounding neuroglial formations. According as they removed from the central bit, the growing ependymal cells changed their shape and lost their ciliae; they thus ceased to be distinguishable from the growing neuroglia and apparently fused with the latter into the same glial or ependymal-glial zone of growth. As in cultures taken from the wall of cerebral ventricles, those areas of the ependymal cover that failed to give extensive growth preserved at the surface of the explants their distinctive features for a rather prolonged period.

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Received
19. VI. 1943.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1943. Volume XLI, № 2

MICROBIOLOGY

KOK-SAGHYZ YELLOWS

By V. L. RISCHKOV

(Communicated by L. A. Orbeli, Member of the Academy, 27. IV. 1943)

Last year the author had received several specimens of kok-saghyz with sharply altered inflorescences. The flowers of these inflorescences were in different stages of antholysis. Their corollas had turned green in colour, the inferior part of the pistils was elongated and the ovule in it had undergone various transformations, which have been described elsewhere in greater detail. Very often such flowers developed a pedicle of their own. Certainly, the plants with flowers thus altered would bear no fruit. There was also observed a further disintegration of the pistil into leaflets, as well as a proliferation of the flower. The pappus of the affected flower goes through different stages of transformation into leaflets, as is usual with *Compositae* affected with antholysis (1). Only the stamina are not involved in this process. They are formed more or less normally, but are sterile in most of the cases. Thus, we have observed in kok-saghyz that type of modification of flowers which we earlier described by the name of stawboor. Along with modified inflorescences the affected plants showed anomalies in vegetative organs as well. Their leaves were chlorotic and developed anthocyan. Anatomical examination has shown that the peduncle of an affected flower-head had its mechanical tissue developed by far more strongly than in the case of a normal flower-head. Also its state of woodiness was more advanced. The general picture formed by all of these characters in the specimens of kok-saghyz here described suggested a virus disease of the yellow type. Later I learned that similar specimens were rather often encountered on the plantations of kok-saghyz in Bashkiria and came myself across plants affected in that way, but these were only occasional instances and up to the present there have been no reports of any extensive spreading of this disease among kok-saghyz plants.

On the assumption that the virus disease in question was more widespread among the weeds and wild-growing plants of Bashkiria, I went into the search for it there, and very soon met with success. I discovered considerable areas overgrown by *Cirsium oleraceum* Scop., on which the plants showed sharp symptoms of affection with the yellow type of virus disease. The affected plants were bushy dwarfs, their leaves were chlorotic and developed anthocyan, their flower-heads exhibited the modifications described above. A similar affection was observed by me in *Cichorium intibus* L. Here again the affected specimens were characterized by stunted growth, chlorosis, developed anthocyan; antholysis and proliferation of flower-heads. In the Botanical Gardens of Ufa I discovered the same disease in *Chrysanthemum aureum* and other species of this genus, as well as in a few specimens of *Sonchus oleraceus* L. and *Taraxacum officinale* Wigg. And in the district of Sterlitamak a specimen of *Matricaria inodora* L. showed similar symptoms. The disease is not confined to *Compositae*; but also occurs among represen-

tatives of other families. Plants affected with it I encountered among the *Valeriana officinalis* L., *Aegopodium podagraria* L.; *Carum carvi* L.; *Plantago major* L., *Cynoglossum officinale* L. In every case the disease manifested itself by the chlorosis which was sometimes preceded by the clearing of veins, and by characteristic modifications in the flower. Anatomical examination always revealed an advanced process of lignification in the stalk and in the peduncle of the inflorescence. In the plants that stored starch the tissue of the leaf abounded in it. In addition to this, stunted growth and bushiness were very often observed. We thus have here the whole complex of symptoms characteristic of the yellow type of virus infection. Attempts to transmit the affection with the juice were not successful, as is generally the case with yellows.

The further rigorous method of proving the infectious nature of the disease is by grafting experiments, but it is not altogether easy on account of certain anatomical and morphological peculiarities of the species liable to such affection.

A flower of kok-saghyz affected with yellow; ovule turned into a leaflet emerging from ovary

Some insect, apparently, is the responsible agent, and the searching for it is the way we have next to follow. Incomplete as are our data on the subject, it may hardly be doubted that we have here to do with a virus disease. Neither fungoid nor bacterial lesions of ètiological significance could be detected in the affected plants. It is also quite certain that the disease is not of the character of a purely entomological injury, since it is a chronic process, and I have observed perennial plants, such as *Chrysanthemum aureum*, in various stages of degeneration caused by the disease. The plants that had been long suffering from the affection bore small slightly yellowish leaves which soon died off, and their shoots were numerous and thin.

We have seen above that the disease occurs among different families. In this respect it is similar to the aster yellows which is due to infection with *Callistephus virus* 1 Kunkel and to which different representatives of the *Compositae*, *Umbelliferae*, *Plantago* sp. and other species and families liable to be affected with the disease in question, are subject. To whatever species belonged the affected representatives investigated by me, the modifications described above were always observed, whereas the aster yellows only in certain cases causes deep changes in the inflorescences. So there is reason to believe that our disease is not the aster yellows. This is also confirmed by the fact that in the Botanical Gardens of Ufa where the affection is very widespread, it never occurs in such typical hosts of *Callistephus virus* 1 Kunkel as *Lactuca*, *Lycopersicum*, *Petunia*; *Callistephus* sp. etc., though they grow there in great numbers and present a diversified set of varieties. Nevertheless, there is considerable likeness between the aster yellows and our disease. In view of its practical and theoretical significance I feel justified to give a brief description of the kok-saghyz yellows before the epiphytology of this disease has been thoroughly studied and its infectuousness rigourously demonstrated.

Though not of a wide occurrence among the kok-saghyz at present, it is nevertheless an extremely harmful disease, as it renders the plant completely sterile. Its further spreading would seriously menace the cultivation of kok-saghyz plants,

because there are such frequent reservatories of the disease as *Cirsium oleraceum*. Moreover, it affects different species of useful and ornamental plants.

From the theoretical point of view this disease is a matter of great interest, because by it our hypothesis that the antholysis and proliferation are a result of the disturbance caused by the delayed outflow of carbohydrates ([2]) is confirmed on a new wealth of material embracing a considerable number of species. Finally, it should be noted that in all of the above-mentioned species susceptible to the affection the retention of carbohydrates was correlated with an enhanced development of the mechanical elements and with lignification. The fact that the reproduction organs are specifically influenced by the deficiency of carbohydrates in the growing point at the time when the flowers are forming, is responsible for the hyperthrophy of the calyx and for different stages of the pistil decomposition into carpels, but affects the formation of the stamina very little with the result that a stawboor type of flower is formed. This regularity which has been noticed by me formerly in representatives of various families suffering from different kinds of virus affection, receives support now from an array of new data.

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Received
27. IV. 1943.

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